

# **INPUT-OUTPUT STRUCTURE, INTERNATIONAL TRADE AND ECONOMIC DEVELOPMENT**

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## **Abstract**

In his paper on the “Structure of Development”, Leontief (1963) claimed that underdeveloped countries are poorer because they are by far less economically diversified. In this paper it is shown that a model of international trade with strong international restrictions on factor mobility, a stable input-output structure, and a productivity externality due to input diversification, is consistent with Leontief’s hypothesis. The model also implies a growth-rate gap between industrialized and less industrialized economies.

JEL classification: F15, F43, O14, O41, O47, R15.

## **Key Words**

Input-Output Structure, Interindustry Linkages, Industrial Diversification, Income Gap, Growth Gap, Migration Restrictions.

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## 1. Introduction

This paper focuses on one of the most conspicuous features of economic development: real *per capita* income is much higher in industrialized nations than in the rest of the world. The World Bank's *World Development Report* shows, year after year, that income gaps among countries do not shrink, especially those between industrialized and less industrialized economies. Analysts who have estimated comparable measures of the countries' product also report such differences (Summers and Heston, 1991). In fact, according to Maddison's (1994) statistical analysis over a sample of 21 countries, the ratio of the highest GDP per capita to the lowest increases from 3.0 to 16.7 between 1820 and 1989 (McGrattan and Schmitz, 1999).

The importance of industrialization for this divergent performance across countries has been pointed out

“Virtually every country that experienced rapid growth of productivity and living standards over the last 200 years has done so by industrializing. Countries that have successfully industrialized –turned to production of manufactures taking advantage of scale economies– are the ones that grew rich, be the 18<sup>th</sup>-century Britain or 20<sup>th</sup>-century Korea and Japan” (Murphy, Shleifer and Vishny, 1989).

This paper also deals with another pattern of economic development concerning not with the income level gap, but the growth-rate gap across countries. This was one of Kaldor's (1961) famous “stylized facts” on economic development: there exists a wide range of cross-country variation in growth performances. For this paper it is interesting to point out that available figures from the World Bank and the Summers and Heston Data Set show that newly industrialized countries are among the 24 highest growing economies in the period 1965-1985 (Barro and Sala-I-Martin, 1995, Table 12.2); all 24 lowest growing economies in the same period are non-industrialized countries or exhibit low level of industrialization (Barro and Sala-I-Martin, Table 12.1); and old industrialized economies, like the OECD countries, exhibit lower growth rates than newly industrialized economies but grow faster than non industrialized economies as a whole.

Providing explanations for these gaps, the level gap and the growth gap, is a goal as old as the science of economics. It goes as far as the formulation of the question about the wealth of nations. Our endeavour is to make a contribution to answer this question within a unified analytical framework. In order to do that, this paper rescues a hypothesis that was first proposed by Wassily Leontief. In “The Structure of Development”, Leontief (1963) claimed that underdeveloped countries are poorer because they are by far less economically diversified.

In this paper it is shown that a model of international trade with strong international restrictions to factor mobility, a stable input-output structure, and a productivity externality due to input diversification, may be consistent with Leontief's viewpoint. Under conditions of sufficiently large differentials in economic diversification, the model delivers an income gap between the industrialized North and the underdeveloped South. This feature is explained, in turn, because the South suffers from an excess of factor supply relative to its low degree of economic diversification. Therefore, given the mobility restrictions, factor remuneration in the South falls with respect to the same remuneration in the North. Southern terms of trade are also deteriorated.

It is convenient to point out that inclusion of perfectly mobile factors within the model does not reverse the income gap result as long as a productive factor experiences international mobility restrictions and this factor is in excess supply in the South.

As mentioned above, an important characteristic of our model is the existence of productivity externalities derived from economic diversification. As in the pin factory of Adam Smith (1776), productivity increases with the social division of labour. The technology embodies the well-known CES utility function of Dixit and Stiglitz (1977) thought of, as in Ethier (1982), as a composite intermediate input that increases with input variety. This specification has been used in well-known endogenous growth models with product diversification: Romer (1987, 1990), Rivera-Batiz and Romer (1991), Grossman and Helpman (1991), Aghion and Howitt (1992), among others.

Therefore, given that the South is characterized by lower economic diversification and endures lower factor remuneration, our model yields lower factor productivity in the South compared with that of the North. This feature has important dynamic consequences. Within the context of an endogenous growth model –Rebelo’s (1991)–, the paper shows that less developed countries tend to grow slower than advanced economies.

In sum, our model predicts that large industrialization disparities between the North and the South not only make less developed countries poorer but also to grow more slowly. Thus, international income gaps tend to grow unless policy makers do something to industrialize the less developed economies (see Aoki et al, 2003).

This paper is organized as follows. The second section contains a brief summary with comments about the basic hypotheses of “The Structure of Development” (Leontief, 1963). The model under autarky is set up in the third section. The fourth section contains the analysis of international trade. Some final comments close this paper in the fifth section.

## 2. The Vision of Leontief

Based on a rigorous analysis of input-output matrices of developed and underdeveloped countries, Leontief finds that the technologies are relatively invariable: each sector exhibits a relatively constant relationship between the inputs it receives from other sectors and its contribution to total product of the economy. According to Leontief, each sector technology is some kind of “recipe” that allows the transformation of some “ingredients” into the sector’s product. As a consequence, the net of interindustry linkages is relatively stable. This technological feature defines the structuralist character of Leontief analysis.<sup>1</sup>

Leontief analysis reveals that the larger and more developed is the economy, the more complete and articulated is its economic structure.<sup>2</sup> Besides, Leontief finds that the most developed economies are structurally quite similar:

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<sup>1</sup> The structuralist vision is common to a well-known group of development economists (see Hirschman, 1958; Chenery and Syrquin, 1975; Chenery, Robinson and Syrquin, 1986; Syrquin and Chenery, 1989; and Syrquin, 1994, among others).

<sup>2</sup> This is a key feature of the process of industrialization which is called input-output deepening: “As countries industrialize, their productive structures become more ‘roundabout’ in the sense that a higher proportion of output is sold to other producers rather than to final users” (Chenery, Robinson and Syrquin, 1986, p. 57). According to these authors, input-output deepening refers to the following related processes: “(...) first, a shift in output mix

“displayed in the input-output table, the pattern of transactions between industries and other major sectors of the system shows that the more developed the economy, the more its internal structures resembles that of other developed economies” (Op. cit., p. 163).

With respect to underdeveloped economies, Leontief observes an important characteristic:

“their input-output tables show that in addition to being smaller and poorer they have internal structures that are different because they are incomplete, compared with the developed economies” (Op. cit., p. 163).

Hence, according to this appraisal, a country’s degree of economic development could be assessed by the relative completeness of its economic structure. Moreover, one may conclude that the more industrialized economies set the technological horizon for underdeveloped economies to reach.

Since a country’s lack of development can be compensated by importation of those goods that it does not produce but needs to consume, Leontief pays special attention to the countries’ profile of international trade. His comparative analyses yield that underdeveloped and developed countries are asymmetrically related in the world markets. The underdeveloped countries are characterized by structural lacks and specialize in primary goods, whilst the developed countries are characterized by structural completeness and specialize in manufacturing products.

According to Leontief, and some other development economists like Chenery et al (1986) and Hirschman (1958), economic diversification itself is an important asset of nations that needs time and effort to be built. That is why nations usually start their process of structural change with sectors that need a shallow use of intermediate goods, and advance step by step, diversifying the economy with sectors whose requirements of intermediate goods are larger.

It is worth mentioning Leontief’s analytical procedure to compare the countries’ economic structures. It implies sorting the sectors in the input-output table according to the degree of backward technological integration.<sup>3</sup> The ensuing triangulation of intermediate transactions reveals the country’s internal economic structure. By means of this procedure, Leontief showed how similar were the internal economic structures of the United States and the developed countries of Western Europe in the sixties. He also revealed, as mentioned above, how incomplete the internal structures of less developed countries were compared with those of more developed countries.

The implication for economic development of this structuralist vision of the economy is straightforward: long-run economic development tends to follow a sequential industrialization process by which inputs are developed first. As in Marx (1867), who believed that more industrialized countries pointed out the path of development to the less progressive

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toward manufacturing and other sectors that use more intermediate inputs; and second, technological changes within a sector that leads to a greater use of intermediate inputs” (idem).

<sup>3</sup> Backward economic integration refers to the relationship between an economic sector and its suppliers of intermediate goods; forward integration refers to the relationship between the sector and its demand sources.

countries, Leontief also claimed that, given the country mix of resources and the available technologies, the essence of the process of development was to create an economic system as similar as possible to the system of the most advanced economies.

Hirschman's theory of "unbalanced" economic growth is directly related to the vision of Leontief on development. Hirschman (1958, 1986), as Leontief, also argued for the possibility of sequential solutions to the process of industrialization. He claimed that investing in those sectors with strong forward and backward technological linkages might bring about economic diversification and economic growth. Hirschman's hypothesis refers to likely paths of industrialization, whilst Leontief hypothesis defines the long-run technological horizon. Leontief and Hirschman hypotheses are thus complementary.

At about the same time Leontief was making his analysis (1963), some countries of Southeast Asia, namely South Korea, Taiwan, Hong-Kong and Singapore, were beginning their industrial takeoffs. As if they were following Leontief advice, their processes of economic development were characterized by quick economic diversification through copying and adapting technologies from developed countries. In a "miraculous" way these previously underdeveloped countries rapidly became industrialized economies. Japan's economic "miracle" started even before World War I; it also followed the same pattern of catching up with the more advanced countries (Amsden 1989; Landes, 1998). Continental China, nowadays, is another example of the same development strategy. Thus, it seems that the wisdom of Leontief deserves some careful consideration.

### 3. The Model in Autarky

The economic structure is represented by an input-output matrix augmented with the vector of capital allocation (see Figure 1). There is no joint production and all sectors are indexed according to its degree of backward technological integration between 0 and  $N$ . The economy is made up of  $N+1$  productive activities:  $N$  intermediate-good sectors and the final good sector.  $X$  represents the vector of intermediate goods produced in the period of analysis; and  $K$  represents the vector of capital. As Figure 1 shows, technological integration is assumed to increase linearly with the sector's index: the sector  $j$  only uses as intermediate inputs the goods with lower index. This feature guarantees that the input-output matrix is perfectly triangular.

As outlined above, some authors have built economic models characterized by input diversity in order to model the effect of productive diversification on the multifactor aggregate productivity of the economy. In these models, however, the technologies are assumed to be equal across intermediate-good sectors. Hence, the triangular structure of the input-output matrix is the main contribution of this paper to the analysis of economic development. This feature is consistent, indeed, with the hypothesis of input-output deepening that was advanced by Leontief (1963) and documented by Chenery et al (1986).

The intermediate inputs of any sector can be read vertically off the input-output matrix. The vector  $Q$ , in particular, is the set of intermediate inputs of the final-good production activity. Notice that the final sector uses in its activity all available intermediate goods.

The technology of the  $j$ -th intermediate good is defined by the following production function

$$(1) \quad X_j = K_j^\alpha \int_0^j X_{ij}^{1-\alpha} di, \quad 0 < \alpha < 1, \quad \forall j \in (0, N),$$

where  $X_j$  is the gross output of good  $j$ ,  $K_j$  is the capital of sector  $j$ , and  $X_{ij}$  is the intermediate consumption of good  $i$  in sector  $j$  ( $i \leq j$ ).

There are some important features of this technology: 1) economic activities are characterized by constant returns to scale in capital and intermediate inputs; 2) intermediate inputs are good substitutes: the marginal rate of technical substitution between any two intermediate inputs is given by  $1/\alpha > 1$  (see Annex 1); 3) all intermediate goods are produced with the same technology, the only difference comes from the range of intermediate inputs used by each sector.

The final good technology is given by

$$(2) \quad Y = K_Y^\alpha \int_0^N Q_i^{1-\alpha} di,$$

where  $Y$  is the final good output,  $K_Y$  is the capital of the sector, and  $Q_i$  is the intermediate consumption of the  $i$ -th input in the final good sector. This technology is then identical to the technology of the  $N$ -th intermediate good. Notice that creation of new inputs –a larger diversification– implies a productivity externality:  $\partial Y / \partial N = K_Y^\alpha Q_N^{1-\alpha} > 0$ . This is the Dixit-Stiglitz (1977) diversification effect.

It is assumed that capital in the period of analysis is given, and it is inelastically supplied. In equilibrium, capital is allocated among the different sectors

$$(3) \quad K = \int_0^N K_j dj + K_Y.$$

Each intermediate good is used in the production of those intermediate goods with higher technological integration. It is also used in the production of the final good. Thus

$$(4) \quad X_i = \int_i^N X_{ij} dj + Q_i, \quad \forall i \in (0, N).$$

Firms in the  $j$ -th sector maximize profits, which are given by the following expression

$$\Pi_j = p_j X_j - r K_j - \int_0^j p_i X_{ij} di.$$

Throughout this paper it is assumed a competitive behaviour in all markets. Hence, given the market prices, the demand for capital and intermediate goods satisfy the following first order conditions for profit maximization

$$(5) \quad K_j = \alpha p_j X_j / r,$$

$$(6) \quad X_{ij} = [(1-\alpha) p_j / p_i]^{1/\alpha} K_j, \quad i \in (0, j).$$

Annex 2 shows the solution for the equilibrium price of the  $i$ -th good

$$(7) \quad p_i = \frac{r}{\alpha \mu_i}, \quad \mu \equiv \alpha (1-\alpha)^{(1-\alpha)/\alpha} > 0, \quad \forall i \in (0, N).$$

Notice that relative prices are fixed:  $p_i/p_j = j/i$ . If the final good is taken as *numeraire*,  $p_Y = p_N = 1$ , the factor price is determined as  $r = \alpha \mu N$ , and the relative price structure is given by

$$(7') \quad p_i = N / i, \quad \forall i \in (0, N).$$

Combination of the price equations (7) and the first order conditions for maximization, equations (5) and (6), yields the technical coefficients for capital and intermediate goods of the  $j$ -th sector

$$(8) \quad \frac{K_j}{X_j} = \frac{1}{\mu j} .$$

$$(9) \quad \frac{X_{ij}}{X_j} = \frac{1-\alpha}{\alpha} \frac{i^{1/\alpha}}{j^{1+1/\alpha}} , \quad \forall i \in (0, j) .$$

Note that technical coefficients in this economy are fixed. This characteristic is not due to the assumption of Leontief technologies –fixed technical coefficients–; actually, as shown above, intermediate goods are assumed to be good substitutes. The fixity of technical coefficients is due to the fixity of relative prices. And this feature, in turn, is due to the assumption of a fixed range of intermediate inputs for each sector. Hence, this model is consistent with Leontief’s finding that technologies are relatively invariable.

The technical coefficients of the final good sector are deduced by symmetry

$$(10) \quad \frac{K_Y}{Y} = \frac{1}{\mu N} .$$

$$(11) \quad \frac{Q_i}{Y} = \frac{1-\alpha}{\alpha} \frac{i^{1/\alpha}}{N^{1+1/\alpha}} , \quad \forall i \in (0, N) .$$

Given the price solutions and the technical coefficients, the gross demand of the  $i$ -th good is deduced. Details are given in the Annex 3. The solution is the following

$$(12) \quad X_i = \frac{1-\alpha}{\alpha} \frac{Y}{N} \frac{i}{N} , \quad \forall i \in (0, N) .$$

Finally, by substituting equations (8), (10) and (12) into equation (3), the capital market equilibrium, the economy’s aggregate production function is deduced:

$$(13) \quad Y = (AN)K , \quad A \equiv \alpha\mu = \alpha^2(1-\alpha)^{(1-\alpha)/\alpha} > 0 .$$

The aggregate production function of this economy implies constant returns to scale with respect to capital. To that extent, it gives some microeconomic foundation to the Rebelo’s (1991) aggregate production function. It is important to note that this production function embodies the Dixit-Stiglitz externalities from economic diversification: aggregate capital productivity,  $AN$ , increases with input diversification ( $N$ ). Hence, the more diversified is the economy the more productive is the final good production. It will be shown later that this feature has important consequences for economic growth.

From the aggregate production function [equation (13)], and the technical coefficient of capital in the final good sector [equation (10)], one deduces the capital allocation to the final good activity

$$(14) \quad K_Y = \alpha K .$$

The remainder,  $(1-\alpha)K$ , is evenly distributed among the intermediate-good sectors. Proof: substitution of equations (12) and (13) into equation (8) yields

$$(15) \quad K_j = (1-\alpha)K / N .$$

## 4. Open Economy

### 4.1. The Basic Set Up

Let us consider the model in the context of international trade. Two economic blocks, the South and the North, are initially in autarky and afterwards they are joined through international trade. Each block is made up of many small countries, so that good prices are competitively determined in the world markets. Some usual assumptions are made: transport costs for goods are assumed to be negligible; and international mobility of capital is prohibited.

It is also assumed that the North has a more diversified economy; i.e. the North produces  $N^*$  goods and the South produces  $N$  goods, such that  $N^* > N > 0$ . From now on all variables related to the North will be denoted with an asterisk. The gap of technological diversification is measured by the ratio  $N^*/N$ . As a consequence, the South and the North are asymmetrically related. Whilst the North may be specialized in goods with higher backward integration ( $i \geq N$ ), it nevertheless can produce the goods with lower backward integration which the South produces. The South, however, cannot produce the higher backward-integrated goods because of its lack of structural diversification.

Hence, under an open trade regime Southern economies give up producing the final good, which is precisely the good with the higher degree of backward technological integration. The proof is straightforward if the factor price is equalized across countries. If the maximum level of diversification is  $N^*$ , and the final good is taken as *numeraire*, the relative price structure is given by  $p_i = N^*/i$  [see equation (7)]. Setting the Northern price for the final good to 1 ( $p_Y = p_{N^*} = 1$ ), the South would be able to produce that good at the price  $N^*/N > 1$ . Hence, the South is driven out of this market, Q.E.D. Equation (7) implies that capital remuneration in the world economy is equal to  $r = \alpha \mu N^*$ . If, on the other hand, the factor price equalization does not hold, one should verify that the North should be able to drive the Southern production out of the final good market –that would be done later–. This condition is necessary for complete specialization of the North in the final good.

It is assumed throughout that no country produces a good if it is not able to produce all the required intermediate inputs. This may seem odd, and so it deserves some explanation. It is easy to understand why inputs have to be developed first in the context of a closed economy: there is no choice. However, in the context of an open economy with small transport costs, it might seem plausible that a country of low industrial diversification could produce a good of higher diversification by importing those inputs the country does not produce. This possibility, however, is not considered by several reasons.

First, the experience of economic development shows that underdeveloped countries follow quite diverse paths of development, but they are restricted to some patterns of structural change (Chenery et al, 1986). Typically, underdeveloped countries start their industrial take off by producing primary goods (Hirschman, 1956). Afterwards they diversify their economic structures by producing intermediate goods which are based mainly in agricultural goods and minerals. In the following stage they develop intermediate goods which use other industrial inputs. Finally they produce capital goods and develop manufacturing goods based on scientific innovations. Along this process the relative size of the service sector expands



permanently, whilst the relative size of the primary sector shrinks. Hence, goods tend to be developed once their inputs are produced.

Second, producing new industrial goods usually implies a previous scientific qualification of economic agents in order to achieve the adoption and adaptation of the specific technologies. Besides, new industrial sectors are usually competitive and feasible only after the accumulation of some experience through learning-by-doing. Hence, it is sensible for developing economies to produce first those goods with a shallow use of intermediates, and advance, step by step, towards economic activities with a longer list of input requirements.

Third, it is well known that transport costs have historically played an important role in the process of industrialization through import substitution. Moreover, even if transport costs are negligible, a near input supplier may imply important strategic advantages for local producers in terms of availability, quickness of delivery and safety against shocks –wars, terms of trade fluctuations, rates of exchange fluctuations and so on–. The argument is advanced by Porter (1990). Hence, it is sensible as well to produce safely first those goods whose inputs are domestically supplied.

The combination of these three arguments is consistent with the experience of industrialization through import substitution, whereby underdeveloped countries industrialize by substituting imported goods for domestic production. This process requires these new sectors to count with domestic intermediate inputs in order to consolidate its economic activity.

It may be worth mentioning that Hirschman theory of “unbalanced” growth considered explicitly investments in some sectors with no domestic supply of inputs. In the mean time, these new sectors had to import their required inputs. His idea, however, was that these investments would create a temporary disequilibrium that would be corrected through import substitution by further investment in the firm’s up-stream suppliers. Similarly, a punctual investment could bring about the development of down-stream firms because of the newly available goods. Hence, economic disequilibrium might exist in the short-run, but they would tend to be corrected in the long run. Hirschman hypothesis may thus depart from Leontief hypothesis in the short-run, but coincide in the long-run period.

#### **4.2. The Factor Price Equalization Might Be Broken**

It will be verified below that, under conditions implying incomplete specialization of the North, the competitive equilibrium of the world economy is analogous to the competitive solution for a closed economy. After all, the world economy is just a bigger closed economy. In this situation the country blocks share some economic activities and the factor price is equalized across countries.

The world capital is simply the sum of Southern and Northern capitals:  $K+K^*$ . If the factor price equalization theorem holds, the allocation of the world capital follows the pattern determined by equations (14) and (15); i.e., the final good activity demands a fraction  $\alpha$  of the world capital:  $K_Y = \alpha(K+K^*)$ , and the remainder is evenly distributed among the intermediate-good sectors:  $K_j = (1-\alpha)(K+K^*)/N^*$ ,  $\forall j \in (0, N^*)$ .

Figure 2 exhibits the cumulative world distribution of capital according to the index of technological integration,  $i$ . The fraction of capital demand for activities with backward

technological integration from 0 to  $N$  is given by  $D(N) = NK_j/(K+K^*) = (1-\alpha)N/N^*$ . Hence, the factor price equalization is sustained as long as the Southern fraction of the world capital,  $K/(K+K^*)$ , is lower than or equal to  $D(N)$ , which implies  $N^*/N \leq (1-\alpha)(1+K^*/K)$ . In this case the South and the North share the production of goods with technological integration lower than  $N$ ; goods with higher technological integration –including the final good– are produced by the North.

Another situation arises if the Southern supply of capital is higher than  $D(N)$ , which implies  $N^*/N > (1-\alpha)(1+K^*/K)$ . In this situation some capital is redundant in the South: the Southern capital supply exceeds the demand for capital in the region. The South ends up completely specialized in those goods with backward technological integration from 0 to  $N$ ; and the North is completely specialized in those goods with higher technological integration –including the final good. Redundant capital would flee to the North if it would be allowed, but it is not, by the assumption of strict international mobility barriers. In the short run some capital from the South may be unused, but in the long run prices tend to adjust, so that the Southern capital remuneration falls with respect to the Northern capital remuneration. At this point an international factor remuneration gap emerges. Moreover, Southern prices are also downwardly adjusted because they are proportional to capital remuneration [see equation (7)]. Southern countries experience, therefore, a deterioration of their terms of trade.

It is important to determine whether this result is sustained if some factors are internationally mobile. In Annex 4 the model is expanded, following Ortiz (1996), to consider two different types of capital. The expanded model reveals that, under the assumption of factor price equalization, both types of capital have a similar cumulative distribution to the allocation distribution of capital shown in Figure 2. Hence, if one of these factors is immobile –let us say, human capital–, and its relative supply in the South is higher than the required demand from activities with backward technological integration between 0 and  $N$ , the factor remuneration falls in the South. Thus, the South experiences a deterioration of terms of trade, and an income gap appears between the North and the South. This result does not change if physical capital –the other form of capital– is perfectly mobile; in this case that mobility ensures the international equalization of physical capital remuneration, but human capital is underpaid in the South.

### 4.3. The Small Country Case with Factor Price Equalization

The case of a Southern country that opens its doors to the world markets when the diversification differential between the South and the North is not large, i.e. when the factor price equalization holds, is analyzed in this section. Figure 3 depicts the situation of this country. It produces with a degree of economic diversification  $N$ . It does not produce the final good so that its whole productive capacity is used to produce intermediate goods within the range  $(0, N)$ . The country produces its own intermediate inputs and the remainder is exported to the rest of the world in exchange for the final good. The export vector is denoted with the letter  $E$ .

Since the technology is analogous to the closed economy case, the country's capital is homogeneously distributed among the  $N$  sectors of activity:  $K_j = K/N$ . It is also easily deduced that factor and input coefficients are fixed. Hence, the gross output of good  $j$  is

deduced by using the equivalent equation (8)

$$(16) \quad X_i = (i/N)\mu K .$$

From the equivalent equation (9) one deduces the intermediate use of the  $i$ -th good in the  $j$ -th sector

$$(17) \quad X_{ij} = (1-\alpha)^{1/\alpha} (i/j)^{1/\alpha} (K/N) , \quad \forall i \in (0, N) .$$

As Figure 3 shows, the exports of the  $i$ -th good are analytically defined as the difference between its production and its intermediate use

$$(18) \quad E_i = X_i - \int_i^N X_{ij} dj = (i/N)^{1/\alpha} \mu K , \quad \forall i \in (0, N) .$$

As shown above, in an integrated world market and with factor price equalization, the relative price structure is given by  $p_i = N^*/i$ . Thus, the export value of the Southern country is given by

$$(19) \quad \int_0^N p_i E_i di = (AN^*)K .$$

In this situation international trade is convenient to the Southern country. Proof: under a closed economy regime, the production of the final good would be equal, as equation (13) shows, to  $(AN)K$ . Specialization in intermediate goods and assuming that factor price equalization holds internationally, implies a welfare gain to the country which starts consuming  $(AN^*)K$  units of the final good through imports. The welfare gain is proportional to the productivity gain of belonging to an international economic system characterized by higher economic diversification ( $N^* > N$ ). Q.E.D.

#### 4.4. International Trade with Complete Specialization

This situation is characterized by Figure 4. The South produces only intermediate goods with backward technological integration from 0 to  $N$ . The North produces those intermediate goods with backward technological integration above  $N$ ; this region also produces the final good. Outputs from the North are denoted with asterisk.

The capital factor from the South is completely allocated to the production of intermediate goods indexed from 0 to  $N$ ; and the capital from the North is completely allocated to the production of the activities of higher backward integration. Notice that goods with backward integration from 0 to  $N$ , which are used in the production activities of the North, are completely supplied by the South. The North exchanges the final good for the intermediate inputs of the South. This situation is consistent with Leontief's appraisal of international trade asymmetries between developed and underdeveloped economies.

Let us examine now the technologies of the North. Equation (20) is the production function of the  $j$ -th good produced by the North ( $X_j^*$ ), which uses as factors of production capital ( $K_j^*$ ), the intermediate goods supplied by the South –those with backward technological integration from 0 to  $N$ –, and the intermediate goods produced by the North itself –those with backward technological integration from  $N$  to  $j$ –

$$(20) \quad X_j^* = (K_j^*)^\alpha \left( \int_0^N X_{ij}^{1-\alpha} di + \int_N^j X_{ij}^{1-\alpha} di \right) , \quad \forall j \in [N, N^*) .$$

Equation (21) is the final good technology, which is identical to the technology of the  $N^*$ -th intermediate good

$$(21) \quad Y^* = (K_Y^*)^\alpha \left( \int_0^N Q_i^{1-\alpha} di + \int_N^{N^*} Q_i^{1-\alpha} di \right).$$

The equilibrium conditions of the goods markets are given by equations (22) and (23). Equation (22) is the equilibrium condition between the exports of the  $i$ -th good from the South and the imports of the same good from the North [ $i \in (0, N)$ ]. These imports are divided between intermediate imports for production of intermediate goods,  $X_{ij}$ , and imports for the final good activity,  $Q_i$ ,

$$(22) \quad E_i = \int_N^{N^*} X_{ij} dj + Q_i, \quad \forall i \in (0, N).$$

Equation (23) represents the equilibrium in the market of the  $i$ -th good produced in the North

$$(23) \quad X_i^* = \int_i^{N^*} X_{ij} dj + Q_i, \quad \forall i \in [N, N^*].$$

It means that supply of the  $i$ -th intermediate good is equated with the demand from the activity of production of intermediate goods and the demand from the final good activity.

The equilibrium condition of the capital market in the North is given by the following equation

$$(24) \quad K^* = \int_N^{N^*} K_j^* dj + K_Y^*.$$

Capital is distributed among the production of  $N^*-N$  intermediate goods and the production of the final good.

The sector  $j$  maximizes profits, which are given by

$$\Pi_j^* = p_j X_j^* - r K_j^* - \int_0^N q_i X_{ij} di - \int_N^{N^*} p_i X_{ij} di.$$

Note that the price of the  $i$ -th intermediate good from the South is denoted with  $q_i$  for  $i \in (0, N)$ ; whilst the Northern prices are denoted with  $p_i$  for  $i \in [N, N^*]$ .

Profit maximization determines that the demand of capital, intermediate inputs of the South, and intermediate inputs of the North satisfy the following first order conditions

$$(25) \quad K_j^* = \alpha p_j X_j^* / r,$$

$$(26) \quad X_{ij} / K_j = [(1-\alpha) p_j / q_i]^{1/\alpha}, \quad \forall i \in (0, N).$$

$$(27) \quad X_{ij} / K_j^* = [(1-\alpha) p_j / p_i]^{1/\alpha}, \quad \forall i \in [N, N^*].$$

First order conditions for profit maximization in the final good activity are deduced by analogy

$$(28) \quad K_Y^* = \alpha p_{N^*} Y / r,$$

$$(29) \quad Q_i / K_Y^* = [(1-\alpha) p_{N^*} / q_i]^{1/\alpha}, \quad \forall i \in (0, N).$$

$$(30) \quad Q_i / K_Y^* = [(1-\alpha) p_{N^*} / p_i]^{1/\alpha}, \quad \forall i \in [N, N^*].$$

The Southern prices are deduced as if the South were a closed economy. Hence, the relative prices of the South should satisfy equation (7), with the difference that capital remuneration in the South is scaled down by the fraction  $\theta$  with respect to capital remuneration in the North ( $r$ )

$$(31) \quad q_i = \frac{\theta r}{\alpha \mu_i}, \quad \forall i \in (0, N), \quad 0 < \theta \leq 1.$$

Using a similar procedure as in section 3, the Northern prices are deduced in Annex 5

$$(32) \quad p_i = \frac{r}{\alpha\mu} [i + (\theta^{\alpha-1} - 1)N]^{-1}, \quad \forall i \in [N, N^*].$$

Note that the price equations (31) and (32) collapse into equation (7) when the factor price,  $r$ , is equalized across countries, i.e. when  $\theta = 1$ .

Now, it is convenient to define the expression between squared brackets in equation (32) as

$$f(i) \equiv i + (\theta^{\alpha-1} - 1)N, \quad \forall i \in [N, N^*].$$

A comparison of the price equations (31) and (32) yields that the price structure is broken when the backward technological index is equal to  $N$ :  $q_N = \theta r / (\alpha\mu N)$ ,  $p_N = \theta^{1-\alpha} r / (\alpha\mu N)$ , and thus  $p_N/q_N = \theta^{-\alpha} > 1$ . This feature is depicted in Figure 5. The smooth, continuous, price structure which is expressed by equation (7) when the factor price equalization holds, is changed by a price structure where the whole set of Southern prices is lower due to a factor remuneration gap between the North and the South. The South experiences a deterioration in terms of trade in order to equilibrate the goods markets and the factor markets. An “unequal exchange” takes place because the productive factor in the South is underpaid.

The next task is to determine the North output. The aggregate production function of the North is deduced in Annex 6

$$(33) \quad Y = \frac{A[f(N^*)]^2}{\alpha f(N^*) + (1-\alpha)(N^* - N)} K^*, \quad A \equiv \alpha\mu = \alpha^2(1-\alpha)^{(1-\alpha)/\alpha} > 0.$$

It is worth noting that specialization of the North increases its productivity and welfare. Proof: If the North had to produce all its required inputs, the aggregate production function would be  $Y = AN^*K^*$ . The above statement is true, then, if the following inequality holds

$$\frac{[f(N^*)]^2}{\alpha f(N^*) + (1-\alpha)(N^* - N)} > N^*.$$

Using the definition of the function  $f(\cdot)$ , the inequality is transformed as follows

$$[(2-\alpha)\theta^{\alpha-1} - 1]N^* + (\theta^{\alpha-1} - 1)^2 N > 0,$$

which is true because all the left-hand side terms are positive. Q.E.D.

Now, the main object of this section is to find an analytical expression for the factor remuneration gap between the North and the South ( $\theta$ ). The Annex 7 yields the following expression

$$(34) \quad \frac{[\alpha + (N^*/N - 1)\theta^{1-\alpha}]\theta^{2-\alpha}}{1 - (1-\theta^{1-\alpha})[1 + (N^*/N - 1)\theta^{1-\alpha}]^{1-1/\alpha}} = (1-\alpha) K^*/K.$$

This is a complex expression. Nevertheless, it can be analyzed as follows. For the discount factor to be a positive fraction ( $0 < \theta < 1$ ), capital per (intermediate-good) sector in the South should be higher than the capital that an economically integrated world would assign to each intermediate-good sector:  $K/N > (1-\alpha)(K+K^*)/N^*$  [or  $N^*/N > (1-\alpha)(1+K^*/K)$ ]. This analysis is based on our knowledge of capital distribution across sectors in a closed economy [see equation (15)]. In other words, there should be an excess factor supply in the South relative to its own degree of industrialization for the existence of a remuneration gap. Equation (34) is

consistent with this analysis because it delivers the limit condition for non existence of excess supply in the South, i.e.  $N^*/N = (1-\alpha)(1+K^*/K)$ , when the discount factor,  $\theta$ , is set equal to 1.

Now, assuming that an excess factor supply does exist in the South, the discount factor diminishes with the industrialization ratio of the North,  $d\theta/d(N^*/N) < 0$ . Figure 6 depicts this behaviour: for an industrialization ratio of the North below or equal to the critical level,  $N^*/N \leq (1-\alpha)(1+K^*/K)$ , the discount factor is 1 (the factor price equalization theorem holds); on the other hand, an industrialization ratio of the North above this critical level implies a gap in factor remuneration between the North and the South –the discount factor is lower than 1–.

Given the determination of the factor remuneration gap, it is possible to determine the aggregate output of the South. By choosing the final good as *numeraire*,

$$p_{N^*} = r [N^* + (\theta^{1-\alpha} - 1)N]^{-1} / (\alpha\mu) = 1,$$

the factor remuneration is determined as  $r = A [N^* + (\theta^{1-\alpha} - 1)N]$ , where  $A \equiv \alpha\mu$ . Hence, given the price structure of Southern goods [equation (31)], and the Southern exports function [equation (18)], it is possible to define the Southern purchasing power in terms of the final good

$$(35) \quad \int_0^N q_i E_i di = \int_0^N \frac{\theta r}{\alpha\mu i} \left(\frac{i}{N}\right)^{1/\alpha} \mu K = r \theta K = A \theta [N^* + (\theta^{\alpha-1} - 1)N] K.$$

From this expression it is possible to conclude that trade also improves welfare in the South with respect to autarky. Proof: the aggregate production function of the South under a closed economy would be  $Y = ANK$ . Hence, the purchasing power in the South is higher under an open economy. For this statement to be true the following inequality should hold

$$A \theta [N^* + (\theta^{\alpha-1} - 1)N] K > ANK,$$

which implies  $N^*/N > 1 + \theta^{-1}(1 - \theta^\alpha)$ . Is this inequality fulfilled under an open trade regime? The answer is positive. In order to be completely specialized in the final good, the North should be able to produce the final good cheaper than the South:  $p_{N^*} < q_N$ . Using the price equations (31) and (32), this price inequality also implies  $N^*/N > 1 + \theta^{-1}(1 - \theta^\alpha) > 1$ . Hence, it must hold. This feature implies a sufficiently large economic diversification between the North and the South.

Finally, the model yields that productivity of capital measured in terms of the final good is higher in the North than in the South. Proof: from equations (33) and (35) one infers

$$\frac{\partial Y}{\partial K^*} = \frac{A[f(N^*)]^2}{\alpha f(N^*) + (1-\alpha)(N^* - N)} > \frac{\partial \left( \int_0^N q_i E_i di \right)}{\partial K} = A \theta [N^* + (\theta^{\alpha-1} - 1)N].$$

Using the definition of the function  $f(\cdot)$ , the previous inequality becomes

$$\frac{N^* + (\theta^{\alpha-1} - 1)N}{N^* + (\alpha \theta^{\alpha-1} - 1)N} > \theta,$$

which is true as the left-hand side expression is higher than 1 (remind that  $\alpha$  is a positive fraction), and  $\theta$  is lower than 1, Q.E.D.

Based on Rebelo's AK model of endogenous growth (Rebelo, 1991), one can postulate that higher productivity of capital implies a higher growth rate. Hence, this model is also consistent with a growth rate gap between developed and underdeveloped countries.

Ortiz (1994) runs some cross-country growth regressions using direct and indirect measures of initial interindustrial dependence; the statistical results do not reject the hypothesis that intermediate-input diversification impinges positively on the rate of economic growth.

Our model is basically static and, thus, it does not consider how economic diversification increases along the path of development. Ortiz (1996, 2002) analyzes some growth models with input-output deepening where, following the hypothesis of Nelson and Phelps (1966), human capital accumulation through education is the key factor to explain the adoption of foreign technologies in the South. This mechanism tends to reduce the international technology gap, but it requires some minimum level of education quality to operate; once this threshold condition is fulfilled, the growth rate increases with education quality. On the other hand, as is well-known, industrialized countries concentrate most of the R&D investment in the world; this feature tends to increase the technology gap. The net effect on the technology ratio,  $N^*/N$ , will depend on the relative strength of the mentioned processes. Most likely, the technology ratio tends to increase and, so, according to our model, growth rates also tend to diverge. This issue deserves further research.

#### 4.5. The Small Country Case with International Income Gaps

Consider the situation when the factor price equalization theorem does not hold and the world economy is characterized by an international gap in per capita real income. This is, of course, the most relevant case. The North is completely specialized in sectors with backward integration higher than  $N$ , and the South is completely specialized in sectors with backward integration from 0 up to  $N$ . In such a case the commercial openness of a small underdeveloped country generates two possibilities:

1) **Low Industrialization.** The country is characterized by a lower industrial diversification than the bulk of underdeveloped economies:  $N^\circ < N$ . From now on the small country is denoted with the superscript  $^\circ$ . In this case, the commercial gains in productivity are at least diminished by the lower Southern prices. The factor remuneration is equalized with that of the Southern countries. Hence, it is not that evident that the small country benefits from a strategy of open markets. Moreover, as international prices are already given, the country may suffer from a low demand to its domestic factor. This is even more worrying if the relative factor endowment of this country is high ( $K^\circ/N^\circ > K/N$ ). In such a case a fraction of the factor may be unused or be driven to informal activities. This is what may be called the “Latin American” Strategy.

2. **High Industrialization.** Our small country is characterized by a higher industrial diversification than the remainder underdeveloped countries:  $N^\circ > N$ . This is what may be named the strategy of the “Asian Tigers”: the small country opens its doors to the world market only when its own degree of economic diversification is higher than the Southern degree of diversification. Then, the small country can produce some intermediate goods that the North produces, those with backward technological integration higher than  $N$ . This country is highly favoured by commercial openness: the country specializes in those goods with higher degree of technological integration, those with technological index between  $N$  and  $N^\circ$ . Therefore, its income increases quickly because produces and exports at the high Northern prices, and buys intermediate goods from the South at low prices. Moreover, its factor

remuneration is equalized with the Northern remuneration, but pecuniary profits enhance the country investment capacity.

In this situation the small country should be able to grow faster than farther industrialized economies. Thus, according to the model, the ordering of economic dynamics in the long-run period replicates the actual data: newly industrialized economies grow quicker than old industrialized economies, and these, in turn, grow faster than low industrialized economies.

The problem with the second strategy is that it cannot be generalized. However, a generalized increment of Southern industrial diversification may lead to lower factor price gaps across countries.

## 5. Concluding Comments

This paper builds a general equilibrium model for the world economy in order to explain why industrialized countries enjoy much higher income levels than underdeveloped countries. The model main features are the following: strong international restrictions to factor mobility, a stable input-output structure, and a productivity externality due to input diversification. Hence, for sufficiently large differentials in intermediate input diversification, the model delivers an income gap between the industrialized North and the underdeveloped South because the factor price equalization theorem does not hold.

The model construction was guided by Leontief's hypothesis that underdeveloped countries are poorer because they are by far less economically diversified. Along this road it was found that the model also helps to explain why newly industrialized countries grow faster than early industrialized countries, and these, in turn, grow faster than less industrialized countries. In order to deduce this ordering, use was made of the new endogenous growth theories: this body of economic literature claims that constant factor productivity with respect to capital factors might imply sustained economic growth, and higher factor productivity implies higher economic growth rates.

From a static point of view it is found that openness is always preferred to autarky: income levels are always higher for an open trade regime independently of the type of economy. However, even from a static point of view, highly industrialized economies benefit more from international trade because their income levels and total factor productivity are higher than in less developed countries. These features of the model help to explain why productive factors from the North do not invade less developed economies, and why leading industrialized countries have been, throughout the history of capitalism, strong advocates of free commercial policies.

From a dynamic point of view, openness is not always preferred because less developed economies may converge to a lower path of economic growth (the "Latin American" strategy). Actually, a less developed economy might be better off under a closed economy regime whilst it builds its economic structure –input diversification takes time–, and then opens its doors to the world market and becomes an exporter of manufacturing goods (the "Asian Tigers" strategy). However, this strategy is not an easy one: it implies a strong policy commitment to diversify the nation's economic structure.

This model shows that international asymmetries in the economic structure of nations are important and, to that extent, history matters: early arrivals to industrialization give a



productive advantage in the world economy. Finally, the paper delivers an important economic policy warning: even under an open trade regime, less developed countries should not indulge themselves and forget the importance of industrial policies for economic development.

### Annex 1: The Elasticity of Substitution between Intermediate Inputs

The marginal productivity of good  $X_{ij}$  is given by

$$\frac{\partial X_j}{\partial X_{ij}} = (1 - \alpha) K_j X_{ij}^{-\alpha} .$$

Cost minimization of good  $X_j$  implies that the ratio of marginal productivities of goods  $X_{ij}$  and  $X_{jj}$  should be equal to the respective price ratio

$$\frac{X_{ij}^{-\alpha}}{X_{jj}^{-\alpha}} = \frac{p_i}{p_j} .$$

Thus, the elasticity of technical substitution between goods  $i$  and  $j$  is given by

$$-\frac{\partial (X_{ij}/X_{jj})}{\partial (p_i/p_j)} \frac{(p_i/p_j)}{(X_{ij}/X_{jj})} = \frac{1}{\alpha} .$$

### Annex 2: The Price Solution (Closed Economy)

Substitution of equations (5) and (6) into equation (1) yields

$$p_j^{-1/\alpha} = (\mu/r) \int_0^j p_i^{1-1/\alpha} di , \quad \mu \equiv \alpha(1-\alpha)^{(1-\alpha)/\alpha} > 0 .$$

By differentiating this expression with respect to  $j$ , one obtains

$$\frac{d p_j}{d j} = -\frac{\alpha \mu}{r} p_j^2 .$$

Integration between 0 and  $i$  yields equation (7) in the text. For this result to follow it is necessary to postulate that the equilibrium price of good 0 is infinity. Proof: given the technology, the production of a good that does not use inputs is zero; hence its only meaningful price is infinity.

### Annex 3: The Output Solution (Closed Economy)

Substitution of equations (9) and (11) in the equilibrium condition of the  $i$ -th market, equation (4), gives the following expression

$$X_i = \frac{1-\alpha}{\alpha} i^{1/\alpha} \left[ \int_i^N \frac{X_j}{j^{1+1/\alpha}} dj + \frac{Y}{N^{1+1/\alpha}} \right] .$$

Differentiating with respect to  $i$  one deduces that  $(dX_i/di)/(i/X_i) = 1$ . Hence,  $X_i = c i$ , where  $c$  is a constant term to be determined. By substituting in the latter expression,  $c$  is identified and the solution for the gross demand of the  $i$ -th good is determined, as in equation (12).

#### Annex 4: Two Capital Goods (Generalizing the Closed Economy Model)

This section is partially based on Ortiz (1996). The technology of each activity is defined by the following production function:

$$(4.1) \quad X_j = K_j^\alpha H_j^\beta \int_0^N X_{ij}^{1-\alpha-\beta} di, \quad 0 < \alpha < 1, \quad 0 < \beta < 1,$$

where  $X_j$  is the gross output of good  $j$ ,  $X_{ij}$  is the intermediate consumption of good  $i$  in sector  $j$  ( $i < j$ ),  $K_j$  is physical capital allocated to sector  $j$ , and  $H_j$  is human capital allocated to sector  $j$ .

The final good technology is identical to the technology of the  $N$ -th intermediate good

$$(4.2) \quad Y = K_Y^\alpha H_Y^\beta \int_0^N Q_i^{1-\alpha-\beta} di.$$

$K_Y$  is the demand of physical capital,  $H_Y$  is the demand of human capital in the final good sector, and  $Q_i$  is the intermediate demand of the  $i$ -th input from the final good sector.

Each intermediate good is used in the production of those intermediate goods with higher technological integration ( $i < j$ ); it is also used in the production of the final good. Thus

$$(4.3) \quad X_i = \int_i^N X_{ij} dj + Q_i, \quad \forall i \in (0, N).$$

In equilibrium physical capital and human capital are allocated among the current sectors

$$(4.4) \quad K = \int_0^N K_j dj + K_Y.$$

$$(4.5) \quad H = \int_0^N H_j dj + H_Y.$$

Profits in the  $j$ -th intermediate-good sector are given by the following expression

$$\Pi_j = p_j X_j - rK_j - wH_j - \int_0^j p_i X_{ij} di.$$

The choice variables are physical capital, human capital and intermediate goods. The corresponding first order conditions for profit maximization are

$$(4.6) \quad K_j = \alpha p_j X_j / r,$$

$$(4.7) \quad H_j = \beta p_j X_j / w,$$

$$(4.8) \quad X_{ij} = [(1 - \alpha - \beta) p_j / p_i]^{1/(\alpha + \beta)} K_j^{\alpha/(\alpha + \beta)} H_j^{\beta/(\alpha + \beta)}, \quad \forall i \in (0, j).$$

Substitution of these conditions into the production function [equation (4.1)] yields the equilibrium price of the  $i$ -th good

$$p_i = \frac{r^{\alpha/(\alpha + \beta)} w^{\beta/(\alpha + \beta)}}{(\alpha + \beta) \mu i}, \quad \mu \equiv [\alpha^\alpha \beta^\beta (1 - \alpha - \beta)^{1 - \alpha - \beta}]^{1/(\alpha + \beta)} > 0.$$

Relative prices are fixed:  $p_i/p_j = j/i$ . By choosing the  $N$ -th good as *numeraire*,  $p_N = 1$ , which implies that everything is measured in terms of the final good, one deduces that the structure of prices is given by the following function

$$(4.9) \quad p_i = N / i.$$

The factor price frontier is also deduced

$$(4.10) \quad r^{\alpha/(\alpha + \beta)} w^{\beta/(\alpha + \beta)} = (\alpha + \beta) \mu N.$$

First order conditions for profit maximization in the final good sector are deduced by symmetry with equations (4.6), (4.7) and (4.8)

$$(4.11) \quad K_Y = \alpha Y / r,$$

$$(4.12) \quad H_Y = \beta Y / w ,$$

$$(4.13) \quad Q_i = [(1 - \alpha - \beta) / p_i]^{1/(\alpha + \beta)} K_Y^{\alpha/(\alpha + \beta)} H_Y^{\beta/(\alpha + \beta)} , \quad \forall i \in (0, N) .$$

Given the price solutions and factor demands, the gross demand of the  $i$ -th good is deduced. Substitution of equations (4.6) to (4.13) into the equilibrium condition of the  $i$ -th intermediate good, equation (4.3), yields the following expression

$$X_i = \frac{1 - \alpha - \beta}{\alpha + \beta} i^{1/(\alpha + \beta)} \left[ \int_0^N \frac{X_j}{j^{1+1/(\alpha + \beta)}} dj + \frac{Y}{N^{1+1/(\alpha + \beta)}} \right] .$$

Differentiating with respect to  $i$  one deduces that  $(dX_i/di)/(i/X_i) = 1$ . Hence,  $X_i = c i$ , where  $c$  is a constant term to be determined. By substituting in the latter expression,  $c$  is identified and the solution for the gross demand of the  $i$ -th good is determined

$$(4.14) \quad X_i = \frac{1 - \alpha - \beta}{\alpha + \beta} \frac{Y}{N} \frac{i}{N} , \quad \forall i \in (0, N) .$$

Using the factor demands [equation (4.6) to (4.8), and (4.11) to (4.13)], the relative price equation of intermediate goods [equation (4.9)], and the market equilibrium of physical an human capital, [equations (4.4) and (4.5)], one deduces the factor prices as functions of the final good production:

$$(4.15) \quad r = \frac{\alpha}{\alpha + \beta} \frac{Y}{K} ,$$

$$(4.16) \quad w = \frac{\beta}{\alpha + \beta} \frac{Y}{H} .$$

Substituting back these equations into the factor-price frontier [equation (4.10)], yields the economy's aggregate production function

$$(4.17) \quad Y = (AN) K^{\alpha/(\alpha + \beta)} H^{\beta/(\alpha + \beta)} , \quad A \equiv (\alpha + \beta)^2 (1 - \alpha - \beta)^{(1 - \alpha - \beta)/(\alpha + \beta)} > 0 .$$

Combination of equations (4.6), (4.9), (4.14) and (4.17) yields the physical capital allocation among the intermediate-good sectors

$$(4.18) \quad K_j = (1 - \alpha - \beta) K / N ,$$

And substitution of equation (4.16) into equation (4.12) yields the allocation of physical capital to the final good activity

$$(4.19) \quad K_Y = (\alpha + \beta) K .$$

Human capital distribution among the intermediate good sectors is deduced by combining equations (4.7), (4.9), (4.14) and (4.17)

$$(4.20) \quad H_j = (1 - \alpha - \beta) H / N ,$$

And the allocation of human capital to the final good activity is deduced by substituting equation (4.15) into equation (4.11)

$$(4.19) \quad H_Y = (\alpha + \beta) H .$$

Hence, a fraction  $\alpha + \beta$  of physical capital is allocated in the final equilibrium to the final good sector, and the remainder, the fraction  $1 - \alpha - \beta$ , is evenly distributed among the  $N$  intermediate good sectors. The same distribution holds for human capital.

**Annex 5: The Price Solution for Northern Prices (Complete Specialization)**

Substitution of the first order conditions for profit maximization of the  $j$ -th sector, equations (25), (26), (27), into equation (20) yields

$$p_j^{-1/\alpha} = (\mu/r) \left[ \int_0^N q_i^{1-1/\alpha} di + \int_N^j p_i^{1-1/\alpha} di \right], \quad \mu \equiv \alpha(1-\alpha)^{(1-\alpha)/\alpha} > 0 .$$

Using equation (31), the previous expression becomes

$$p_j^{-1/\alpha} = (\alpha\mu N/r)^{1/\alpha} + (\mu/r) \left[ \int_N^j p_i^{1-1/\alpha} di \right].$$

Differentiating with respect to  $j$

$$\frac{dp_j}{dj} = -\frac{\alpha\mu}{r} p_j^2 .$$

Integrating between 0 and  $i$ , one deduces equation (32).

**Annex 6: The Aggregate Production Function of the North (Complete Specialization)**

Substitution of equations (25), (28) and (32) into equation (24) yields

$$(\bullet) \quad K^* = \mu^{-1} \int_N^{N^*} \frac{X_j^*}{j + (\theta^{\alpha-1} - 1)N} dj + \mu^{-1} \frac{Y}{N^* + (\theta^{\alpha-1} - 1)N} .$$

Differentiating with respect to  $j$

$$\frac{dK^*}{dj} = 0 = \mu^{-1} \int_N^{N^*} \frac{[j + (\theta^{\alpha-1} - 1)N] dX_j^*/dj - X_j^*}{[j + (\theta^{\alpha-1} - 1)N]^2} dj .$$

Hence,

$$\frac{dX_j^*}{dj} = \frac{X_j^*}{j + (\theta^{\alpha-1} - 1)N} .$$

Integrating between 0 and  $i$ , and solving, yields the relative output of the  $i$ -th intermediate good

$$(\bullet\bullet) \quad \frac{X_i^*}{X_N^*} = \frac{\theta^{1-\alpha}}{N} [i + (\theta^{\alpha-1} - 1)N] = \frac{\theta^{1-\alpha}}{N} f(i) , \quad \forall i \in [N, N^*] .$$

Substituting back into equation ( $\bullet$ ), for  $X_j$ , yields

$$(\bullet\bullet\bullet) \quad K^* = \frac{X_N^*}{\mu\theta^{\alpha-1}} \left( \frac{N^*}{N} - 1 \right) + \frac{Y}{\mu f(N^*)} .$$

Now, substitution of equations (25), (27), (28), (30) and (32) into equation (23) yields:

$$\begin{aligned} X_i^* &= \frac{1-\alpha}{\alpha} [f(i)]^{1/\alpha} \left[ \theta^{1-\alpha} X_N^* N^{-1} \int_i^{N^*} [f(j)]^{-1/\alpha} dj + u^{-1} [f(N^*)]^{-(1+1/\alpha)} Y \right] \\ &= \frac{\theta^{1-\alpha} X_N^*}{N} f(i) \left\{ 1 - \frac{[f(N^*)]^{1-1/\alpha}}{[f(i)]^{1-1/\alpha}} \right\} + \frac{1-\alpha}{\alpha} \frac{[f(i)]^{1/\alpha} Y}{[f(N^*)]^{1+1/\alpha}} . \end{aligned}$$

Using equation ( $\bullet\bullet$ ) one deduces

$$(****) \quad X_N^* = \frac{1-\alpha}{\alpha} \frac{N}{[f(N^*)]^2} \frac{Y}{\theta^{1-\alpha}}.$$

Combining equations (\*\*) and (\*\*\*\*) yields the aggregate production function of the North [equation (33) in the text].

### Annex 7: The Remuneration Gap between The North and the South

Substitution of equations (25), (26), (28), (29), (31), (32) and (\*\*) into equation (22) yields

$$\begin{aligned} E_i &= \frac{1-\alpha}{\alpha} (i/\theta)^{1/\alpha} \left[ \frac{X_N^*}{N} \int_N^{N^*} [f(j)]^{-1/\alpha} dj + \frac{Y}{[f(N^*)]^{1+1/\alpha}} \right] \\ &= \frac{1-\alpha}{\alpha} (i/\theta)^{1/\alpha} \left[ \frac{X_N^*}{N} \frac{\alpha}{1-\alpha} \{ [f(N)]^{1-1/\alpha} - [f(N^*)]^{1-1/\alpha} \} + \frac{Y}{[f(N^*)]^{1+1/\alpha}} \right] \end{aligned}$$

Substitution of equations (18), (33) and (\*\*\*\*) into the previous expression, yields equation (34).

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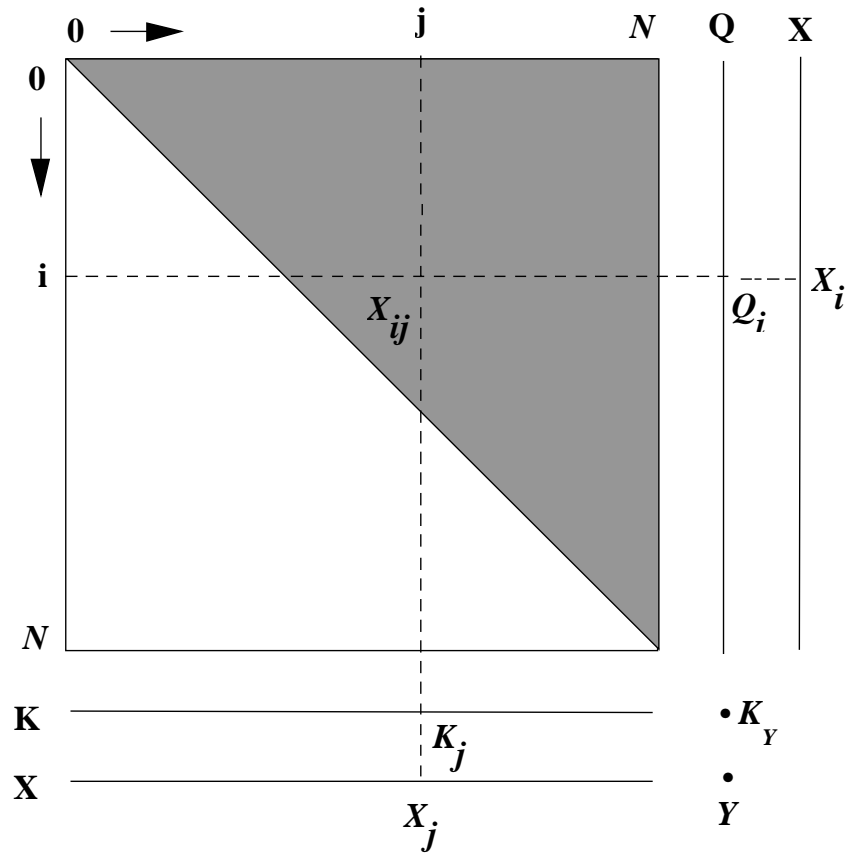
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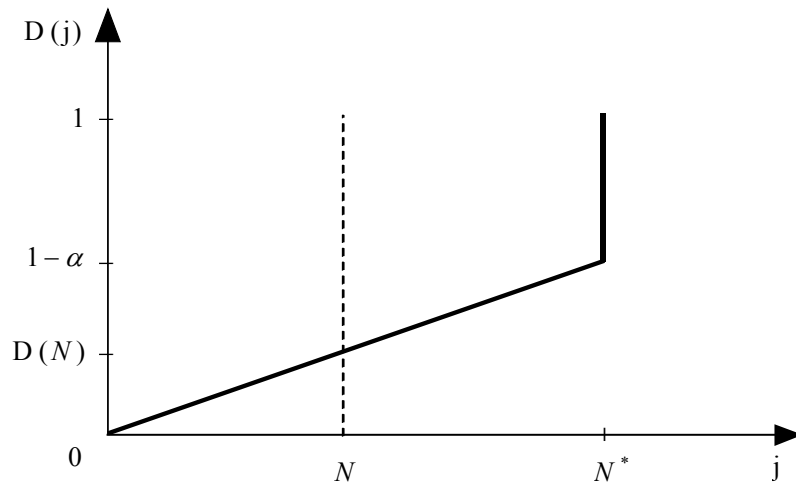
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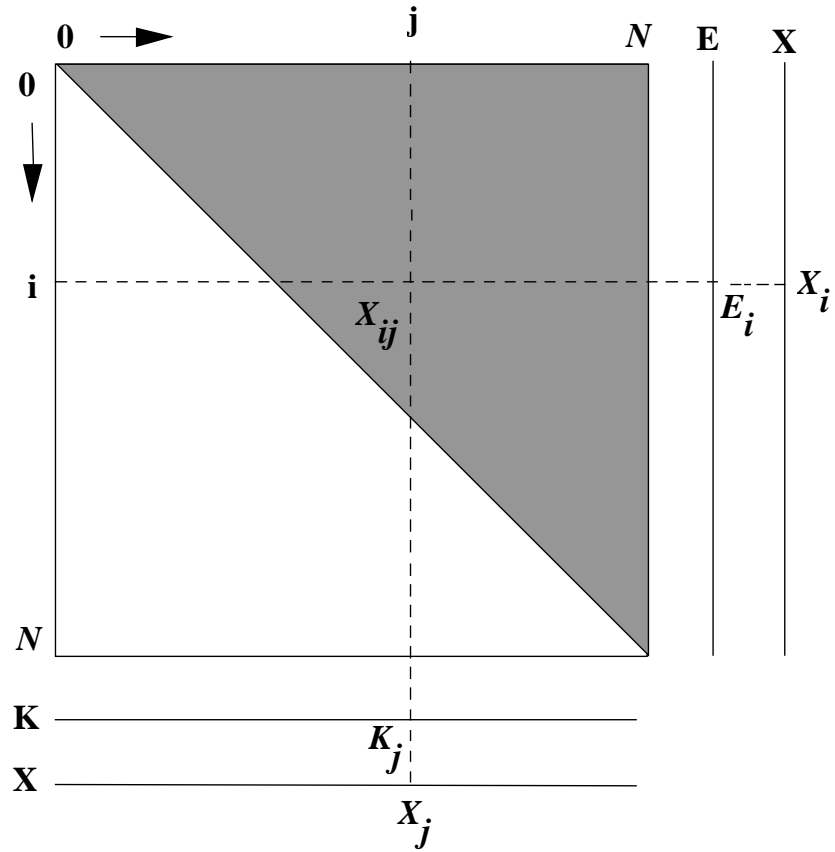
Figure 1  
Closed Economy Structure



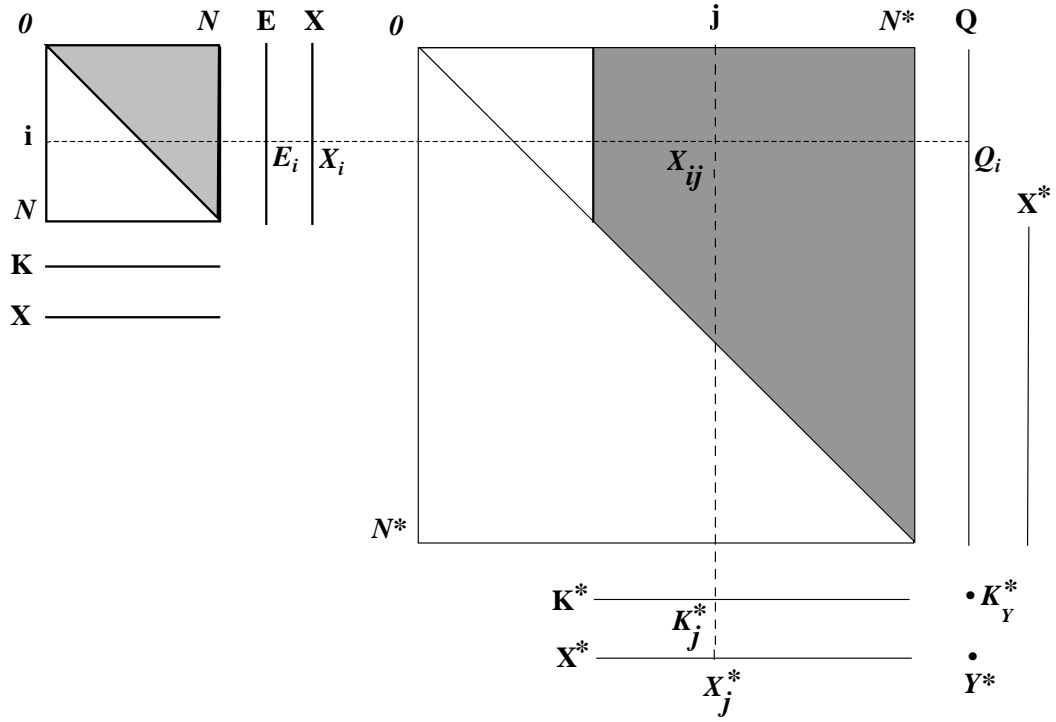
**Figure 2**  
**Capital Distribution in the World Economy**



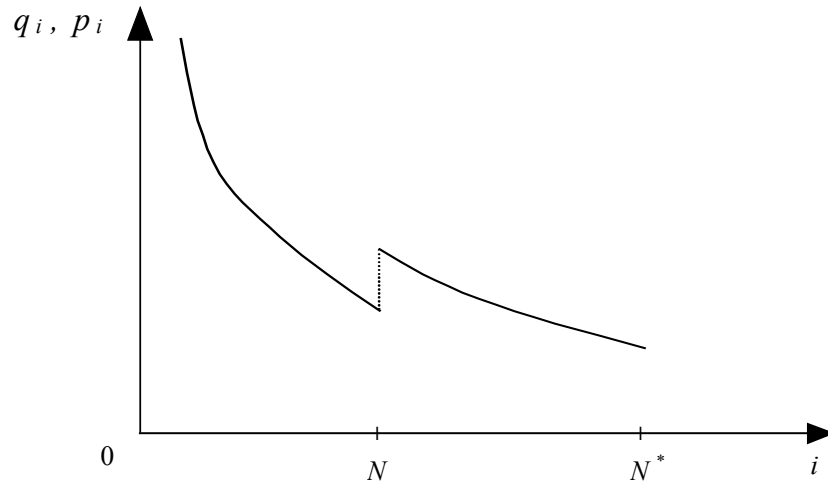
**Figure 3**  
**Open Economy: The Small Country Case**



**Figure 4**  
**World Economy: South and North**  
**Complete Specialization**



**Figure 5**  
**Price Structure**



**Figure 6**  
**Factor Remuneration Gap between the North and the South**

