

**ECONOMIC STRUCTURE,  
EDUCATION AND GROWTH**

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## ECONOMIC STRUCTURE, EDUCATION AND GROWTH<sup>1</sup>

By Carlos H. Ortiz

**Abstract:** This paper analyzes the effects of education on the process of economic diversification in underdeveloped countries. Under the assumption that most technological changes are driven by imitation, we describe how a process of input-output deepening evolves and how this process increases real income.

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## 1 INTRODUCTION

A casual examination of any country's input-output matrix shows different degrees of backward technological integration across sectors: some activities require more inputs and some activities require less. Similarly there exist different degrees of forward integration: some sectors providing more intermediate inputs than final goods, and some sectors providing intermediate goods serve more activities than others. This structure of interindustry linkages is usually very stable even when the economy is shocked by strong changes in relative prices. Hence, it is natural to think that sectors with higher backward technological integration should develop later than those sectors with lower backward integration. This intuition is right: one of the more robust features of economic development is the evolution of the economic structure towards more technologically integrated forms of production (Leontief, 1963). This process is reflected in an increasingly complicated structure of interindustry linkages. Following Chenery, Syrquin and Robinson (1986), we shall call this process input-output deepening.

Along the path of development the economy enjoys a wider availability of goods that reflects an increasing process of social division of work. If division of work increases labour productivity, as in the pin factory of Adam Smith, we may have social gains in productivity coming from the multiplication of economic activities across the society. Romer (1987, 1990), Grosman and Helpman (1991) and others have explored this intuition and have shown the possibility of sustained growth in models where technological progress increases permanently the social division of labour. However, they do not capture the phenomenon of input-output deepening as they assume *ab initio* identical technologies for all inputs of a final good technology. Besides, these models have focused on the process of technological change through innovation. Hence they are more suitable to the analysis of economic development in industrialized economies where the possibilities of increasing the range of available goods (and the degree of social division of work) come basically from technological innovation.

However, for developing countries the main source of economic diversification is the copy, transfer and adaptation of existing technologies from developed countries. This is not to deny the possibility of important technological breakthroughs in developing countries, but clearly the non-rival character of technological information and also the limited possibility of excluding developing countries from using the technologies previously discovered in developed countries, make it cheaper and more advantageous for developing countries to become specialized in copying existing technologies.

Since we are primarily interested in modelling the economics of developing countries, our model will be based on technological change through copy and adaptation. We will ignore the existence of patents and assume that adaptation of technologies can be done by investing in know-how. We will also consider an economic structure that experiences a growing degree of backward technological integration among sectors. This assumption plays an important role in our model because the learning process is then subject to the sequential order that the "deepening" of the technologies imposes.

In the first stage of our research we will analyze a closed economy where economic diversification is brought about by technological transfers. It may seem odd to assume both autarky in trade and the possibility of technological transfers. However, such a scenario arises naturally where there is initially a high degree of protection due to transport costs or prohibitive tariffs. This means that the nature of foreign goods is known, but their consumption is restricted until the country starts its own production. In other words, we will assume that technology transfer is much cheaper than transfer of goods.

Technological transfers are, however, limited by the process of accumulation of human capital. In our model human capital is interpreted as the knowledge of a given number of technologies: we may understand the technology as a “recipe”, one for each good, that allows the transformation of some “ingredients” into new goods<sup>2</sup>. Before starting cooking the chefs must learn the recipes. Hence increasing human capital (i.e. learning new recipes) depends on the quality and efficiency of education and the allocation of some effort. Thus, the transfer and adaptation of technology is a process that requires continuous education of the country’s workforce.

In our model, as in Lucas (1988), education is a condition for improvement of human capital. But education diverts resources from productive activities. If no effort is allocated to education, the range of goods (and sectors) is unchanged but the current level of output is maximum. On the contrary, if the whole workforce is allocated to education, the growth of human capital is maximum (the learning rate of recipes is maximum), but output is zero. Hence there exists a trade-off between education and production.

Now, once a recipe is learned it stays with us forever. Furthermore, as we learn recipes we get to know more ingredients and then it is easier to learn even more recipes. Hence, we will assume that the technology of education is linear in the current level of human capital (the number of recipes known) and the amount of effort allocated to education. This linearity is the source of sustained diversification in our economy.

The paper is organized as follows. In section 2 we develop the model in autarky. Section 3 extends this model to consider the effects of international trade.

## **2 A MODEL OF ECONOMIC DIVERSIFICATION THROUGH EDUCATION**

### **2.1 The Model in Autarky**

The economic structure is represented instantaneously by an input-output matrix augmented with the vector of workforce allocation (see Figure 1). There is no joint production and all sectors (and goods) are indexed according to the degree of backward technological integration between  $\theta$  and  $N$ . This integration is assumed to increase linearly with the sector’s index: the sector  $j$  only uses as intermediate inputs the goods with lower index. This feature guarantees that the input-output matrix is perfectly triangular. The

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<sup>2</sup> The idea is taken from Leontief (1963).

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intermediate inputs of any sector can be read vertically off the input-output matrix. The labour force is indexed according to its allocation among sectors.

The technology of each activity is defined by a modified Cobb-Douglas production function:

$$(1) \quad X_j = AL_j^a \int_0^j X_{ij}^{1-a} di,$$

where  $X_j$  is the gross output of good  $j$ ,  $A$  is a technological parameter,  $X_{ij}$  is the intermediate consumption of good  $i$  in sector  $j$  ( $i < j$ ), and  $L_j$  is the workforce allocated to sector  $j$ . The technology is characterized by constant returns to scale and perfect substitutability among intermediate inputs. Equation (1) implies that all goods are produced with the same technology, the only difference comes from the size of the range of intermediate inputs used by each sector.

At any given moment in time a fraction  $m$  of the labour force is offered inelastically:

$$(2) \quad \int_0^N L_j dj = m,$$

where  $N$  measures the current range of existing goods. The labour force is assumed to be constant and normalized to 1.

All goods are perishable and all of them are suitable for final consumption. Hence, the gross demand of good  $i$  is made up of intermediate demands and final consumption:

$$(3) \quad X_i = \int_i^N X_{ij} dj + c_i,$$

where  $C_i$  is the final demand for good  $i$ . Notice that the  $i$ -th sector is integrated forward only with sectors of higher backward integration ( $X_{ij} > \theta$  for  $i < j$ ;  $X_{ij} = \theta$  for  $i \geq j$ ).

We will assume that the representative consumer derives utility from the consumption of any good and maximizes the discounted stream of utility over an infinite horizon. The objective function is defined as follows:

$$(4) \quad \int_0^N e^{-pt} u(\{C_i(t)\}) dt,$$

where  $p$  is the discount rate,  $u(\bullet)$  is the instantaneous utility function and  $\{C_i(t)\}$  is the vector of current final consumption over the range  $[\theta, N(t)]$ .

In order to complete the characterization of instantaneous equilibrium we require a specification for instantaneous preferences. We will assume the following modified constant elasticity of substitution utility function:

$$(5) \quad \begin{cases} \left( \frac{\left( \int_0^N C_i^r di \right)^{(1-e^{-1})r^{-1}} - 1}{1 - e^{-1}} \right), \text{ for } e > 0, \neq 1, \mathbf{g} > 0, \\ r^{-1} \ln \left( \int_0^N C_i^r di \right), \text{ for } e = 1, \mathbf{g} > 0, \end{cases}$$

where  $\mathbf{e}$  is the intertemporal elasticity of substitution of the given bundle of goods, and a  $0 \leq [1/(1-\gamma)]$  is the (instantaneous) intra-temporal elasticity of substitution among goods. Although the orthodox CES function is usually assumed to be homogeneous of degree 1 ( $\mathbf{e}^{-1} = \theta$ ), we assume the utility function to be strictly concave ( $\mathbf{e}^{-1} > \theta$ ) with a high intertemporal elasticity of substitution ( $\theta < \mathbf{e}^{-1} \leq 1$ , or  $\mathbf{e} \geq 1$ ). These functional forms imply that the representative consumer experiences diminishing marginal utility with respect to any given bundle of goods. This assumption ensures an interior solution to the dynamic path. We also assume a high intratemporal elasticity of substitution among goods ( $\theta < \gamma < 1$ , or  $\sigma > 1$ ). This last assumption is necessary for a positive marginal utility from diversification ( $\gamma > \theta$ ),<sup>3</sup> and also for obtaining well-behaved demand functions for individual goods.

The previous equations complete the static model. Before characterizing the corresponding equilibrium, we proceed to define the technology of human capital accumulation. This will provide the dynamics of our model.

Human capital is simply the accumulated knowledge of technologies defined by the number of existing sectors (goods):  $N(t)$ . We assume that our economy's human capital is small compared to more advanced economies. We also assume that technological knowledge is non-excludable. Hence, our economy specializes in appropriating foreign technologies. However, this process requires educated agents. Furthermore, the appropriation of new technologies requires new skills. Hence, the process of economic diversification continues as long as the agents allocate some effort to education. Since knowledge is not subject to depreciation, the technology of education is defined by the following function:

$$(6) \quad \dot{N}(t) = N(t) [1 - m(t)] \mathbf{d}$$

where a dot denotes a time derivative. Thus the rate of creation of new sectors (goods) is proportional to the current level of knowledge,  $N(t)$ , and the amount of effort allocated to education as measured by the fraction of workforce which is not working,  $1 - m(t)$ . The parameter  $\mathbf{d}$  is an index of productivity in education.

Given the possibility of education the agents in this economy face an intertemporal trade-off: it pays to invest in education today -working less and producing a lower output- in order to enjoy a broader range of goods tomorrow. This assumes, of course, that the productivity in education is sufficiently high: the rate of diversification of goods must be

<sup>3</sup> It can be checked that  $u(\bullet) / N > \theta$  if  $\gamma > \theta$ .

sufficiently high in order to compensate for the lower level of current consumption. Additionally, for an interior solution of the dynamic problem, we need the instantaneous utility function to be concave in its arguments, namely the set of goods currently available. That is why we assume a high intertemporal elasticity of substitution ( $\epsilon \geq 1$ ).

## 2.2 The Instantaneous Equilibrium

The representative consumer maximizes his instantaneous utility, equation (5), subject to the instantaneous budget constraint which is defined by the following expression:

$$(7) \quad \int_0^N P_i C_i di = mw$$

where  $w$  is the wage rate,  $mw$  is current income, and  $p_i$  is the (unit) price of good  $i$ .

The consumer takes as given income and prices, generating the following relative demand function:

$$(8) \quad \frac{C_i}{C_j} = \left( \frac{P_i}{P_j} \right)^{-s}, \quad s = \frac{1}{1-\epsilon} > 1$$

Note that the inequality  $\tilde{\alpha} < 1$ , or  $\tilde{\alpha} > 1$ , guarantees that relative demands fall with relative prices.

Firms' profits in sector  $j$  are defined as follows:

$$\mathbf{p}_j = P_j X_j - wL_j - \int_0^j P_i X_{ij} di$$

Due to the assumption of constant returns to scale one can aggregate the firms in each sector. In order to maximize profits, firms in sector  $j$  choose the amount of labour force to be hired and the intermediate inputs from the range  $[\theta, j]$ . The factor demands are calculated assuming the wage and the input prices as given. The first order conditions for this problem are the following:

$$(9) \quad L_j = \mathbf{a} P_j X_j / w,$$

and

$$(10) \quad X_{ij} = [(1-\mathbf{a})AP_j / P_i]^{1/\mathbf{a}} L_j, i \in [0, J]$$

Now we can straightforwardly calculate the price of each good. Substitution of equations (9) and (10) into equation (1) yields



$$(11) \quad P_j^{-1/a} = \frac{a}{w} \int_0^j P_i^{1-1/a} di, a \equiv \left[ \mathbf{a}^a (1-\mathbf{a})^{1-a} A \right]^{1/a} > 0$$

Differentiating with respect to  $j$  gives

$$\frac{dp_j}{dj} = -\frac{\mathbf{a}}{w} a P_j^2$$

Integrating between  $\theta$  and  $i$  we find

$$(12) \quad P_i = \frac{W}{\mathbf{a} a i}$$

We are able to obtain such a simple equation for the relative price of good  $i$  because under the technological assumptions, see equation (1), the output of good  $\theta$  is zero: non integrated sectors do not produce output, hence the only meaningful price of good zero is infinity<sup>4</sup>. Equation (12) shows that the relative prices decrease asymptotically towards zero with the degree of backward technological integration.

Given the structure of relative prices we can solve for the technical coefficients. Substitution of equation (12) into equation (9) yields the technical coefficient for labour in sector  $j$ :

$$(13) \quad \frac{L_j}{X_j} = \frac{1}{aj}$$

Now, by combining equations (10), (12) and (13) we obtain the intermediate input coefficients of sector  $j$ :

$$\frac{X_{ij}}{X_j} = \frac{1-\mathbf{e}}{\mathbf{a}} \frac{i^{1/a}}{j^{1+1/a}} \quad \text{for all } i \in [0, J]$$

The last two equations show that given the degree of technological integration,  $j$ , the technical coefficients are “fixed” as in a Leontief technology. Note, however, that we do not assume fixed technological coefficients. Actually, intermediate inputs in each activity are assumed to be perfect substitutes [see equation (1)]. Fixed technological coefficients in this model are due to fixed relative prices. Thus, in our economy the workers learn only one way of making each good and the “recipes” are never modified (not even in composition).

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<sup>4</sup> The technology may be modified allowing sector zero to be autonomous, e. g.

$$X_j = AL_j^a \left( \int_0^j X_{ij}^{1-a} di + X_{0j}^{1-a} \right), \text{ so that } X_0 = AL_0^a X_{00}^{1-a}$$

This technology would yield a positive output of good zero that would be supported by a positive price. However, it does not seem that we gain much by complicating the model in this way.

Let us solve now for the final demand for good  $i$ . By combining equations (7), (8) and (12) we deduce

$$(15) \quad C_i = a a^s m \left( \frac{i}{N} \right)^s$$

This equation shows that the final demand structure is biased in favour of sectors with high backward technological integration ( $i$  close to  $N$ ). This result is not surprising as relative prices fall with the *degree* of backward integration [see equation (12)]. The bias in the final demand structure is stronger the higher the intratemporal elasticity of substitution. The structure of final demand is illustrated in Figure 2.

Equation (15) also implies that the final demand structure shifts in favour of newer goods as the number of sectors increases. Thus the final demand for sectors with a low degree of backward technological integration ( $i \approx \theta$ ) becomes negligible. Again, the higher is the elasticity of substitution the stronger is this effect.

Let us solve now for the structure of gross demands. Substitution of equations (14) and (15) into equation (3) yields

$$(16) \quad X_i = \frac{1-a}{a} i^{1/a} \int_i^N \frac{X_j}{j^{1+1/a}} dj + \frac{a a^s m}{N^s} i^s$$

Differentiating twice with respect to  $i$  yields

$$\frac{d^2 X_i}{di^2} = \frac{a a^s (a a^s - 1)}{N^s} i^{s-2}$$

This is a second order differential equation whose general solution has the form  $X_i = \phi_0 + \phi_1 i + \phi_2 i^\sigma$ , where,  $\phi_0$ ,  $\phi_1$  and  $\phi_2$  are constant coefficients to be determined. By substituting this solution into equation (16) we can identify these coefficients and obtain the solution for the gross demand of good  $i$ :

$$(17) \quad X_i = \frac{a a^s}{a^s - 1} \left[ \left( 1 - a \left( \frac{i}{N} \right)^s \right) m \right]$$

From this equation we deduce that the economic structure profile depends on the relationship between the elasticity of intratemporal substitution in final consumption,  $\sigma$ , and the output elasticity of labour,  $\sigma$ . Figure 2 shows the possible shapes of the gross demand structure. The economic intuition for these shapes is as follows.

The final demand always increases with the degree of backward economic integration,  $i$ , because highly integrated sectors produce cheaper goods. Given “fixed” technological coefficients [see equation (14)], the gross demand tends to increase with final demand. However, the bias of the final demand structure towards highly integrated goods needs not determine the bias of the gross demand structure: even if the final demand for lower

integrated goods is negligible, they are still required as intermediate inputs in the production of highly integrated sectors. These derived demands will be higher the larger is the intensity of intermediate input in the production technology, i.e. the lower  $\alpha$ . Thus, if the bias toward final goods is not too high (the elasticity of substitution,  $\sigma$ , is not too high), and production is intensive in intermediate goods ( $\alpha$  low), so that  $\alpha\sigma < 1$ , the gross demand may be biased towards sectors with an intermediate degree of technological integration. This case is illustrated in Figure 2a. On the other hand, high elasticity of substitution and/or low production intensity in intermediates, so that  $\alpha\sigma > 1$ , determine a bias in gross demand towards highly integrated sectors. This case is illustrated in Figure 2c. Figure 2b illustrates the borderline case.

Now, by combining equations (13) and (17) we deduce the labour demand in sector  $j$ :

$$(18) \quad L_j = \frac{s}{s-1} \left[ (1-a) + (as-1) \left( \frac{j}{N} \right)^{s-1} \right] \frac{m}{N}$$

Figure 3 shows the different possibilities of labour allocation across sectors. The structure of employment is clearly related to the structure of gross demand. Even sectors with the lowest backward technological integration are demanded at least as intermediate inputs. Thus they require some allocation of labour. If production intensity in intermediates is high, the labour demand is biased towards sectors with low technological integration (the labour profile is downward sloping); if production intensity in intermediates is low, the labour demand is biased towards sectors with high technological integration; in the borderline case all sectors hire identical number of workers.

Given the structure of final demand we can solve for the instantaneous level of utility. For simplicity we will choose the case of logarithmic preferences ( $\epsilon=1$ ). However, in the Appendix we show that our main results are not significantly changed by allowing for a higher degree of intertemporal substitution. Now, plugging equation (15) into equation (5), for  $\epsilon=1$ , yields

$$(19) \quad u = \ln \left[ b m N^{s/(s-1)} \right] \quad b \equiv a a^{-1/(s-1)} O$$

Hence, the instantaneous level of utility depends on the fraction of labour force allocated to productive activity ( $m$ ), and the range of existing goods in the economy ( $N$ ). Equation (19) shows why it is natural to assume a high degree of intratemporal substitutability among goods ( $\sigma > 1$ ): only in this case the society's welfare increases with the range of available goods,  $N$ .

### 2.3 The Dynamic Equilibrium

The consumer maximizes equation (4) subject to the instantaneous utility function, [equation (19)] and the transition equation of education [equation (6)]. The Hamiltonian equation associated with this problem is

$$H(\dots) = \text{Max} \left\{ \ln \left[ \mathbf{b}m(t) \quad N(t)^{\frac{\mathbf{s}}{\mathbf{s}-1}} \right] e^{-\rho t} + \mathbf{I}(t)N(t)[1 - m(t)]\mathbf{d} \right\},$$

where the arguments of the Hamiltonian are  $m(t)$ ,  $N(t)$  and the multiplier  $\lambda(t)$ .

The first order conditions for maximization are

$$(20) \quad H_m(\dots) = 0 : e^{-\rho t} m(t)^{-1} = \mathbf{d}\mathbf{l}(t)N(t)$$

and

$$(21) \quad \begin{aligned} \dot{\mathbf{I}}(t) &= -H_N(\dots) \\ &= -\left( \frac{\mathbf{s}}{\mathbf{s}-1} N(t)^{-1} \mathbf{q}^{-\rho t} + \mathbf{I}(t)[1 - m(t)]\mathbf{d} \right) \end{aligned}$$

The equilibrium path of this economy should satisfy the following transversality condition:

$$(22) \quad \lim_{t \rightarrow \infty} \mathbf{I}(t) N(t) = 0$$

Now we proceed to find the equilibrium. By combining equations (20) and (21) we obtain

$$\frac{\dot{\mathbf{I}}}{\mathbf{I}} = -\mathbf{d} \left( 1 + \frac{m(t)}{\mathbf{s}-1} \right)$$

By differentiating equation (20) with respect to time, and using the last equation and equation (6), we deduce the differential equation that drives workforce allocation:

$$\frac{\dot{m}(t)}{m(t)} = -\mathbf{r} + \frac{\mathbf{d}\mathbf{s}}{\mathbf{s}-1} m(t)$$

The phase picture corresponding to this equation is in Figure 4.

Rest points are  $m(t) = \theta$ , and the following steady state equilibrium:

$$(23) \quad m^* = \frac{\mathbf{r}\mathbf{s}-1}{\mathbf{d}\mathbf{s}}$$

Under the assumption of interior solution,  $m^*$  is the only solution consistent with the transversality condition. Hence there is no transitional dynamics in this model, i.e. forward-looking agents choose at once the level of labour supply  $m^*$  given by equation (23).

With logarithmic preferences ( $\epsilon = 1$ ), and a high elasticity of intratemporal substitution among goods ( $\sigma > 1$ ), the workforce allocation to productive activities is always positive ( $m^* > \theta$ ). On the other hand, the allocation of time to education might be positive ( $m^* < 1$ ), if the following inequality holds:  $\delta > \rho (\sigma - 1) / \sigma$ . This means that given some degree of impatience,  $\rho > \theta$ , the workforce will get educated if the degree of intratemporal substitutability among goods is high and the education system is sufficiently efficient. If the last condition does not hold, i.e.  $\delta > \rho (\sigma - 1) / \sigma$ , no time is allocated to education<sup>5</sup>.

This analysis implies a relationship between labour supply, education efficiency and welfare gains. Refer to Figure 5. Below the threshold level of efficiency in education no education takes place and hence economic diversification does not progress. For high levels of education efficiency, some effort is allocated to education (the labour supply is lower), but the number of sectors increases at the following rate:

$$(24) \quad \frac{\dot{N}}{N} = \mathbf{d} - \frac{\mathbf{s} - 1}{\mathbf{s}} \mathbf{r},$$

and the utility level increases permanently:

$$(25) \quad \dot{u} = \frac{\mathbf{s}\mathbf{d}}{\mathbf{s} - 1} - \mathbf{r}$$

These results define the trade-off between education and labour supply.

At this point we should note that equation (25) implies that the growth rate of welfare gains ( $\dot{u}/u$ ) falls steadily towards zero. This is a consequence of assuming an intertemporal elasticity of substitution equal to one ( $\epsilon = 1$ ); however, if this elasticity is larger than one ( $\epsilon > 1$ ), the growth rate of welfare gains falls asymptotically towards the following positive minimum (see the Appendix):

$$(25^*) \quad \lim_{t \rightarrow \infty} \frac{\dot{u}(t)}{u(t)} = (\mathbf{e} - 1) \left( \frac{\mathbf{s}\mathbf{d}}{\mathbf{s} - 1} - \mathbf{r} \right)$$

The intertemporal elasticity of substitution measures the willingness to postpone consumption today for consumption tomorrow. Thus a higher elasticity reflects a propensity to allocate a higher level of effort in education, which yields a higher rate of welfare growth.

Finally, if an interior solution exists the transversality condition boils down to the requirement that the discount factor be positive,  $\rho > \theta$ .

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<sup>5</sup> A rigorous deduction of this result implies restricting our Hamiltonian equation to solutions for  $m \leq 1$ . This procedure however is reduced to yield a Kuhn-Tucker multiplier equal to zero (and so  $m = 1$ ) for education efficiency lower than or equal to the threshold level identified in the text.

## 2.4 Theory and facts

The model developed so far yields endogenous welfare growth. This is due exclusively to diversification since output in each sector decreases steadily as the fixed amount of labour is allocated among an increasing number of activities [see equation (18)]. However, the model may be helpful to understand some facts of the process of industrialization.

First of all, as the Figure 5 shows, the model is able to explain why some countries follow a path of industrialization and others stagnate. A country diversifies its economic structure if education efficiency is above some minimum level, so that private agents find optimum to invest in education.

Secondly, given a process of diversification, the rate of welfare growth falls over time. This feature may be helpful to explain why newly industrialized countries tend to grow at very high rates (Stern, 1989). It is also consistent with the hypothesis of convergence as tested by many empirical studies (Barro, 1991; Barro and Sala-i-Martin, 1992; Mankiw, Romer and Weil, 1992).

Thirdly, along the path of development the demand structure shifts in favour of new goods which are characterized by high degree of backward technological integration. It is important to point out that for some degree of output diversification the model exhibits negligible consumption of those goods with the lowest degree of backward integration. These features are consistent with the structural transformation associated with the process of industrialization (Chenery et al, 1986).

## 3 THE OPEN ECONOMY CASE

Consider our model of economic diversification in the context of international trade. Refer to Figure 6. Two economies, the South and the North, are initially in autarky and afterwards they are joined through international trade. The population is mobile within the countries but international migration is prohibited. The single factor that can be accumulated is human capital, which is here the same as the cumulative knowledge of technologies. We will assume that the North owns a higher level of human capital and thus has a more diversified economy; i.e. the South produces  $N$  goods and the North produces  $N^*$  goods, such that  $N^* > N > \theta$ . From now on all variables related to the North will be starred.

For relative prices we obtain the same solutions as in the closed economy case because they are determined solely by the degree of backward technological integration of each sector [see equation (12)]. With international trade the prices of identical goods are equalized. With common production activities, the factor price equalization theorem implies that the wage rate is identical in the South and the North [see again equation (12)].

Let us now turn to the determination of the world gross demands. As Figure 6 shows the world demands are given by

$$(26) \quad \begin{aligned} X_i^w &= X_i + X_i^* = \int_i^N X_{ij} dj + \int_i^{N^*} X_{ij}^* dj + C_i + C_i^*, i \leq N, \\ X_i^w &= X_i^* = \int_i^{N^*} X_{ij}^* dj + C_i + C_i^*, N \leq i \leq N^*, \end{aligned}$$

where the superscript W denotes world demand. Note that the equilibrium condition for goods within the range  $[N, N^*]$  includes only the intermediate demands of the North as we know that the South does not produce this range of goods. But we should include the final demand for these goods from the South as well as those from the North.

Let us note briefly the fundamental asymmetric relationship between the South and the North. Whilst the North may be specialized in those sectors with higher backward integration, it nevertheless can produce the goods with lower backward integration which the South produces. However, the South cannot produce the higher backward integrated goods because of its lack of human capital.

Next we need to obtain expressions for the constituents parts of equations (26). We will start with the final demands. All consumers share the same utility function [see equation (5)], and all of them have access to the consumption of  $N^*$  goods. This means that the South can consume goods it does not produce through international exchange.

The final demands are given by the following formulas which are equivalent to equation (15):

$$(27) \quad \begin{aligned} C_i &= \mathbf{a} \ a \ \mathbf{s} \left( i/N^* \right)^{\mathbf{s}} \ m L \\ C_i^* &= \mathbf{a} \ a \ \mathbf{s} \left( i/N^* \right)^{\mathbf{s}} \ m^* L^*, \end{aligned}$$

where  $m$  is the fraction of the workforce in productive activities in the South, and  $L$  is the workforce in the South. Starred variables again correspond to the North.

Now, labour demand and intermediate inputs are proportional to the gross output in each sector, as we saw in the previous section [see equations (13) and (14)].

Substituting these demands into equations (26) we solve for the gross demands. The result is the following:

$$(28) \quad X_i^w = \frac{a\mathbf{s}}{\mathbf{s}-1} \left[ (1-\mathbf{a}) \left( \frac{i}{N^*} \right) + (\mathbf{a}\mathbf{s}-1) \left( \frac{i}{N^*} \right)^{\mathbf{s}} \right] (mL + m^* L^*),$$

which is analogous to the solution in the closed economy case [see equation (18)]. Equation (28) applies to all goods within the range  $[\theta, N^*]$ . Hence, there are no discontinuities in the world demand structure at the level of the  $N$ -th good, as one might believe by looking at Figure 6. The intuition for this feature is that the world final demand structure is smooth. Hence, given “fixed” intermediate input coefficients, the gross demand structure should be

smooth as well. For a graphical intuition of this result it may be helpful to add the input-output matrices as well as the vectors of labour and gross product in Figure 6. Thus, we are back to the “closed economy” case, and the smoothness of the world demand structure follows.

Given the solution for the world gross demands we can solve for the world demand for labour in industry  $j$  by using equation (13):

$$(29) \quad L_j^w = \frac{\mathbf{s}}{\mathbf{s}-1} \left[ (1-\mathbf{a}) + (\mathbf{a}\mathbf{s}-1) \left( \frac{j}{N^*} \right)^{\mathbf{s}-1} \right] \frac{mL + m^*L^*}{N^*}$$

Then, integrating between 0 and  $N$  and dividing by the world labour demand,  $mL + m^*L^*$ , we deduce

$$(30) \quad d\left(\frac{N}{N^*}\right) = \frac{\mathbf{s}}{\mathbf{s}-1} \left[ (1-\mathbf{a}) \left(\frac{N}{N^*}\right) + \frac{\mathbf{a}\mathbf{s}-1}{\mathbf{s}} \left(\frac{N}{N^*}\right)^{\mathbf{s}} \right],$$

which is the fraction of the world labour demand in the range of activities  $[\theta, N]$ . Note that this fraction increases with the relative level of human capital in the South,  $N/N^*$ . The line OET in Figure 7 depicts the fraction of world labour demand corresponding to activities with a degree of backward integration lower than  $i$ ,  $i \in (\theta, N^*)$ . In drawing this line we assume that  $\alpha\sigma > 1$ , so that employment demand increases more than proportionally with the degree of backward integration [see Figure 3]. However, the important issue is that the labour demand line OET is increasing in  $i$  for the relevant case of high degree of intratemporal substitutability ( $\sigma > 1$ ).

Now, if the fraction of labour supply corresponding to the South is denoted by  $s$  ( $= mL/(mL+m^*L^*)$ ), we have three possibilities (refer again to Figure 7):

(1) If the South supplies the fraction of labour  $s_1$ , the North employs a fraction of its workforce equal to the ratio  $AE/AZ$  in activities with backward integration lower than  $N$ , the remainder, given by the ratio  $EZ/AZ$ , is employed in activities with higher backward integration. The actual distribution of output supply in common activities (with backward integration lower than  $N$ ) is not determined.

(2) If the South happens to supply the fraction of labour  $s_2$ , it will be specialized in activities with backward integration lower than  $N$ . The North, of course, will be specialized in activities with higher backward integration. In this case there will be only one activity in common, the marginal activity with backward integration equal to  $N$ .

(3) If the South provides the fraction of labour supply  $s_3$ , the southern wage will fall in order to correct the excess supply of labour given by the distance  $EC$ .



In the last case the factor price equalization theorem does not apply because the South will be completely specialized in products with backward integration lower than  $N$ . Because relative prices are proportional to wages [see equation (12)], the Southern goods' prices fall relative to the Northern goods' prices.

Figure 7 shows that, given a high relative supply of labour force in the South as  $s$ , the excess supply,  $EC$ , is larger the lower the level of human capital in the South,  $N$ , relative to the level of human capital in the North,  $N^*$ . This follows from the fact that the relative demand line,  $OET$ , is increasing in the degree of backward integration. Thus the wage adjustment is stronger the lower is  $N/N^*$ .

These results may help to explain why countries with low school enrolment ratios -which are usually taken as good proxies for human capital accumulation- have less diversified economies and low real incomes. Evidence on the correlation between growth and school enrolment ratios is found in Barro (1989a, 1989b). Evidence of the relationship between economic structure and real income is found in Leontief (1963), Chenery, Robinson and Syrquin (1986), and Syrquin and Chenery (1989).

Figure 7 also exhibits an important property: if the relative supply of labour from the South is high, so that the Southern wage is below the Northern wage, the South might increase its real income by increasing its human capital level relative to the human capital level of the North. This would be the case of a successful process of industrialization. In the early stages of this process the South is compelled to compete with low wages. However, if the process of human capital accumulation in the South is sufficiently rapid -which implies a quick process of import substitution and input-output deepening-, wages and prices should be increasing until the point in which they equalize Northern wages and prices. At this stage the South will enter the "natural" market of the North, i.e. the South will start producing goods which were previously produced only by the North. After this event the process of technological diversification may be pursued through education and learning but surely technological innovation will become a necessary condition for industrialization to be sustained.

The experience of newly industrialized countries is consistent with the preceding analysis. As a general rule they adopted policies for export promotion, but experienced first an early process of import substitution -sometimes supported by protectionist policies (Stern, 1989). They also diversified very quickly its productive structure (Chenery, Robinson and Syrquin, 1986) and set high standards of education efficiency (World Bank, 1991). The result was a quick process of industrialization based on learning and adaptation of foreign technologies (see Amsden, 1989, for the Korean experience; for a general description of the industrialization process see Stern, 1989, and the 1991 World Development Report).

In this paper we have emphasized the importance of education for economic growth. We assumed throughout that education efficiency is a structural parameter. However, if the government has some influence on establishing standards of education, this model predicts that a country is more likely to start a process of diversification and welfare growth by setting high standards of education.

### Appendix: Generalizing the Model for a High Degree of Intertemporal Substitutability

In section 2 we assumed an intertemporal elasticity of substitution equal to 1. Here we show that the main results of that section are not affected provided that this elasticity is high, i.e.  $\varepsilon > 1$  [see equation (5)].

Substitution of equation (15) into equation (5), for  $\varepsilon > 1$ , yields the instantaneous level of utility:

$$u(m, N) = \frac{1}{1-\varepsilon^{-1}} \left\{ \left[ \mathbf{a} \mathbf{a} \mathbf{s}^{1/(1-s)} m N^{s/(s-1)} \right]^{1-s^{-1}} - 1 \right\}$$

For  $\varepsilon = 1$ , the last equation collapses to our equation (19). Now, as before we want to maximize the discounted sum of utility [equation (4)], subject to the transition equation of education [equation (6)]. The first order conditions for this problem are the following:

$$e^{-rt} \frac{\partial u(m(t), N(t))}{\partial m(t)} = \mathbf{I}(t) N(t) \mathbf{d},$$

$$\frac{\dot{\mathbf{i}}}{\mathbf{I}} = -\mathbf{d} \left[ 1 + \frac{m(t)}{\mathbf{s} - 1} \right],$$

$$\lim \mathbf{I}(t) N(t) = 0$$

where  $\mathbf{i}$  is the shadow value of the stock of knowledge.

Following the same procedure as in section 2 we deduce the workforce allocation to productive activities:

$$m = \mathbf{e} \left( \frac{\mathbf{s} - 1}{\mathbf{s}} \frac{\mathbf{r}}{\mathbf{d}} - 1 + \mathbf{e}^{-1} \right)$$

Afterwards we obtain the rate of utility growth:

$$\frac{\dot{u}}{u} = \frac{(1 - \mathbf{e}^{-1}) \mathbf{s}}{\mathbf{s} - 1} \frac{\mathbf{a} \mathbf{a} \mathbf{s}^{1/(1-s)} m N^{(1-s^{-1})s/(s-1)}}{\mathbf{a} \mathbf{a} \mathbf{s}^{1/(1-s)} m N^{(1-s^{-1})s/(s-1)} - 1} \frac{\dot{N}}{N},$$

where

$$\frac{\dot{N}}{N} = \ell \left( \mathbf{d} - \frac{\mathbf{s} - 1}{\mathbf{s}} \mathbf{r} \right)$$

hence the conclusions of section 3.2 are valid here. Notice also that as the number of

sectors grows, the growth rate of utility falls towards the minimum value shown in equation (25').

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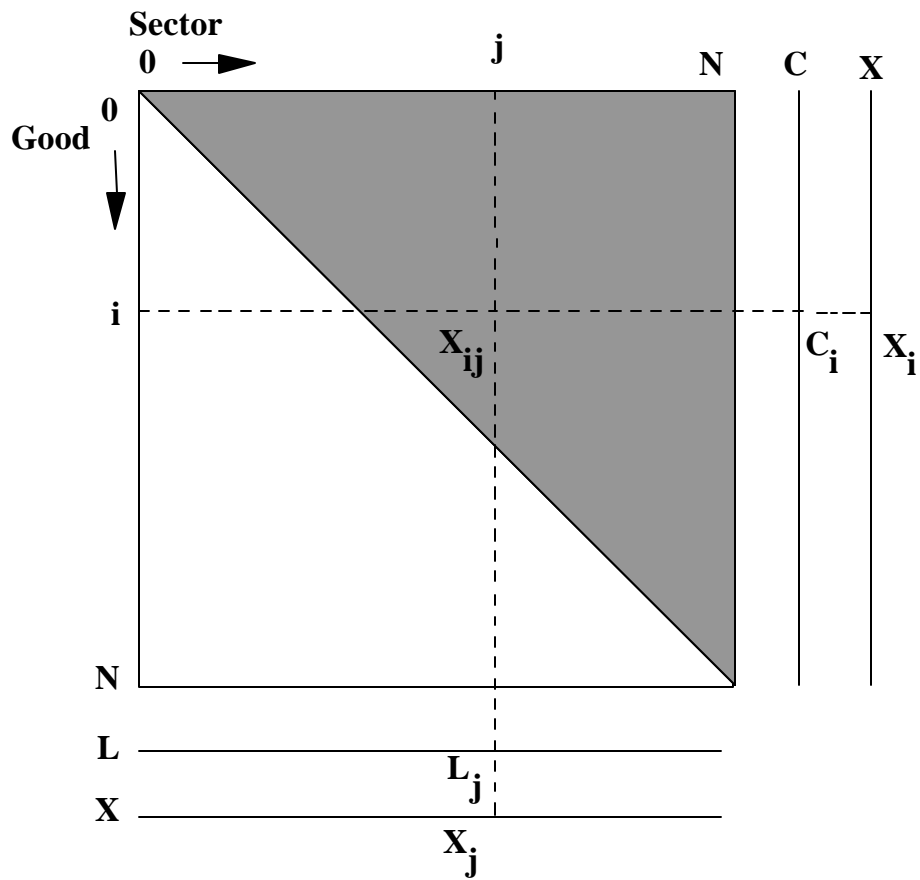
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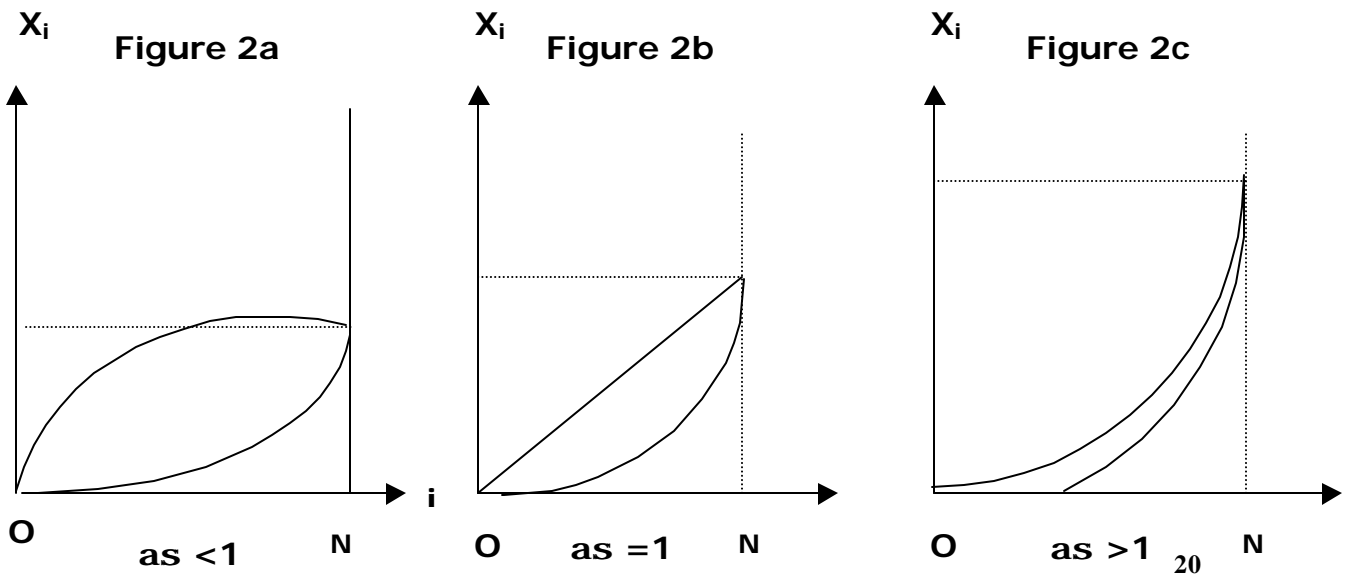
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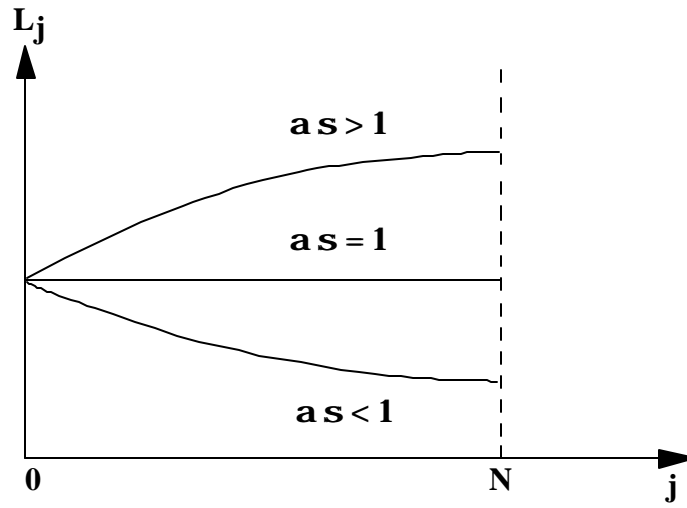
**Figure 1**  
**INPUT-OUTPUT MATRIX**



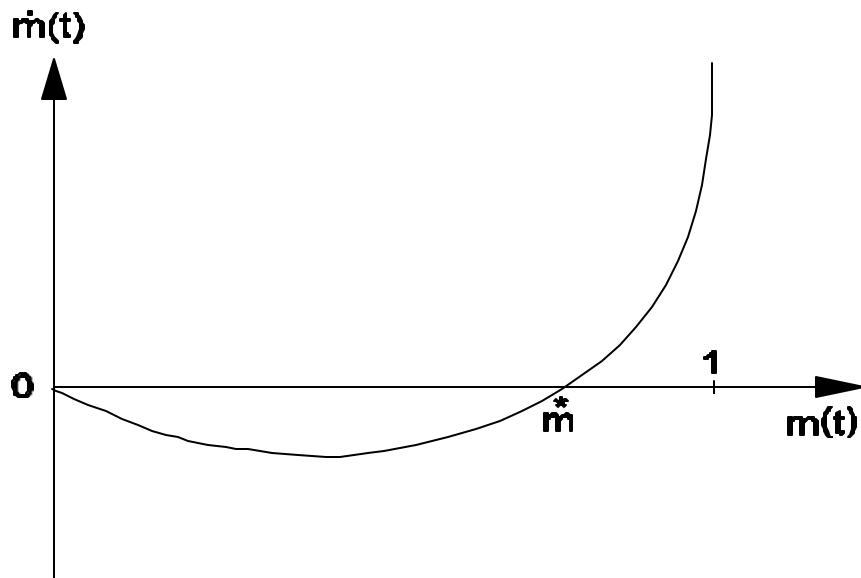
**Figure 2**  
**DEMAND STRUCTURE**



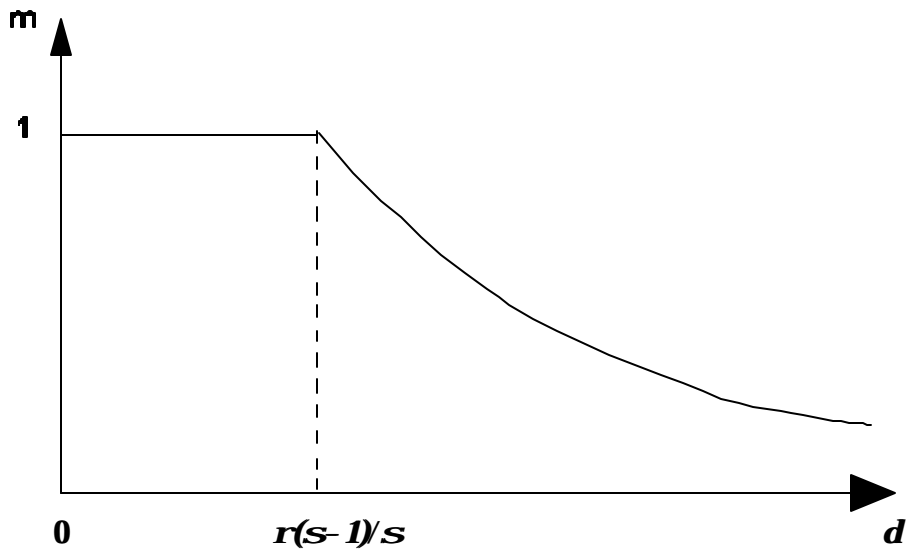
**Figure 3**  
**EMPLOYMENT STRUCTURE**



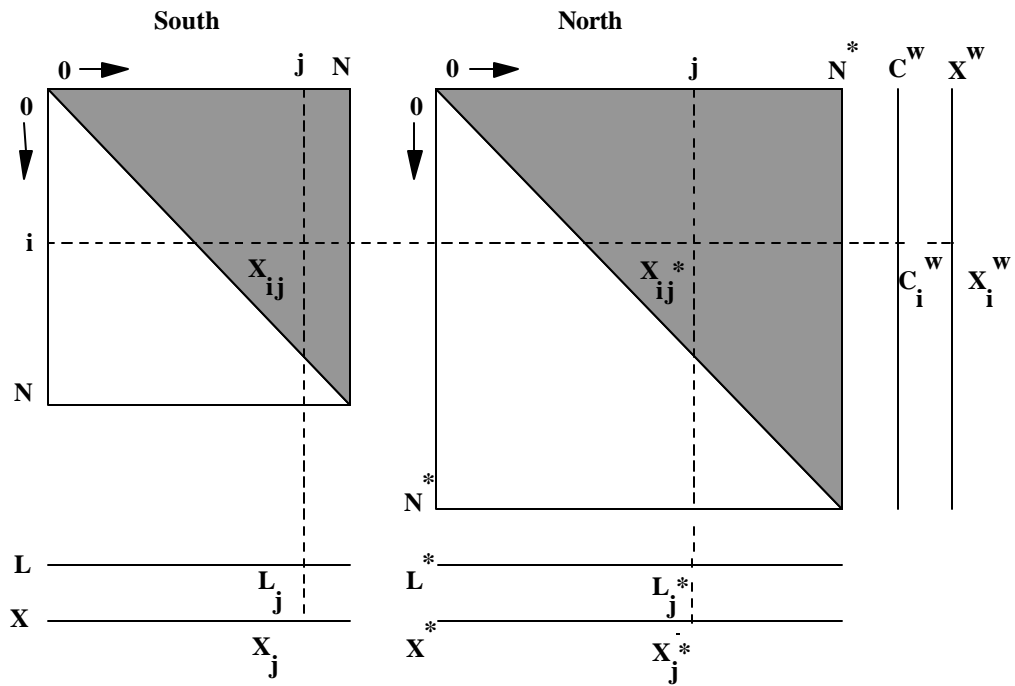
**Figure 4**  
**LABOUR FORCE DYNAMICS**



**Figure 5**  
**LABOUR SUPPLY**



**Figure 6**  
**INPUT-OUTPUT MATRICES**



**Figure 7**  
**LABOUR ALLOCATION**

