Dinámicas distributivas de la satisfacción vital en Europa

Distributional dynamics of life satisfaction in Europe

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Resumen.
En este trabajo se analiza la evolución de la desigualdad y la polarización en la satisfacción vital en el grupo de países Euro-12. Nos basamos en la literatura reciente para emplear medidas que respeten la naturaleza ordinal de esta variable. Se destaca la distinción teórica de estos dos fenómenos de la distribución, caracterizando el índice propuesto por Abul Naga y Yalcin (2008) como un indicador de polarización para un rango específico de los parámetros. Nuestra principal conclusión es que la desigualdad en satisfacción vital era significativamente mayor en 2014 que en 1995. El aumento a nivel agregado se explica básicamente por el incremento de la desigualdad en los países mediterráneos y de Irlanda. La polarización en satisfacción vital muestra un patrón similar al de la desigualdad, aunque se observan algunas diferencias reseñables.
Palabras clave. Europa, satisfacción vital, desigualdad, polarización, variables ordinales, Gran recesión.

Abstract. In this paper, we analyze the evolution of inequality and polarization in life satisfaction in the Euro-12 group of countries. We rely on the recent literature to employ measures that respect the ordinal nature of this variable. An especial emphasis is made in the theoretical distinction of these two distributional phenomena, characterizing the Abul Naga and Yalcin (2008) index as a measure of polarization for a particular range of the parameters. Our main finding is that life satisfaction inequality was significantly higher in 2014 than in 1995. The rise at the aggregate level is basically explained by the increase in inequality in the Mediterranean countries and Ireland. Polarization in life satisfaction exhibits a similar pattern to that of inequality, albeit some significant departures are observed.
Key words. Europe, life satisfaction, inequality, polarization, ordinal variables, Great recession.

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Índice

1. Introduction

The distribution of life satisfaction is gaining ground as public policy issue in Europe. Article 2 of the Treaty on European Union (consolidated version 2012) establishes as one of the aims of the Union the promotion of "the well-being of its peoples". As regards the definition of well-being, the Communication from the European Commission to the Council and the European Parliament (2009): "GDP and beyond: Measuring progress in a changing world" advocates overcoming restricted approaches to well-being, and among other aspects it considers life satisfaction one of its relevant dimensions.[1] Furthermore, the same Communication highlights the importance of attending to distributional aspects as social and economic cohesion is considered to have both intrinsic (from a normative point of view) and instrumental (as a condition for sustainable social progress) importance.

Precisely, previous literature addressing the distribution of subjective well-being has motivated its research referring to its normative and practical implications. Stevenson and Wolfers (2008) note that, dealing with subjective well-being, normative implications are somewhat limited. In particular, assuming the non existence of diminishing marginal returns as regard this good, there would not be such implications for a committed utilitarian because no transfer would improve average well-being. In any case, the issue remains relevant for all normative theories concerned with distributional justice (Ott, 2005; Madden, 2011; Dutta and Foster, 2013). As regards the instrumental value of social and economic cohesion and its relation with the distribution of subjective well-being it is widely held that happiness inequality is an indicator of social tensions (Veenhoven, 2005; Becchetti et al., 2014), or of the need for change (Sachs, 2012). As regards the particular case of the European Union, and, as have been seen, in line with its own statements in several documents, Apergis and Georgellis (2015) assert that "happiness convergence could also be a legitimate policy goal and a useful barometer of changes in the political landscape, societal values, and citizens' sentiments about further European Integration" (Apergis and Georgellis, 2015, p. 68).

There is scant evidence on the evolution of the distribution of life satisfaction in Europe. Veenhoven (2005, 2011) studied the evolution of its standard deviation for the period 1973-2010 by means of the Eurobarometer surveys and observed that it has declined in all countries but West Germany and Portugal. The reduction in life satisfaction inequality has been especially remarkable in the Mediterranean countries (Greece, Italy, Spain). As regards the observed increase in life satisfaction inequality in Germany, the finding is consistent with new evidence found by Becchetti et al. (2014) who use the Germany Socio-Economic Panel database to study changes in the distribution of life satisfaction along the period 1991-2007 in Germany. They find that the increase in inequality is mainly due to changes occurred in the lower tail of the life satisfaction distribution.

[1] It is now well accepted that well-being in a country is more than just its GDP per capita (Alkire, 2002). As regards the role of subjective well-being, it has been always advocated by the utilitarian tradition (Dutta and Foster, 2013), which influences, partly through Stiglitz *et al.* (2009), main current quality of life measurement initiatives (UNDP 2010; OECD 2011; ESS 2011). Nevertheless, several caveats have been pointed out with respect to the usage of subjective well-being self-assessments to inform about how we are faring both individually and socially and to evaluate alternative states of affairs. For this reason we would devote next section to explain main features of subjective well-being data and discuss further its usefulness for well-being evaluations.

In this paper we also use the Eurobarometer surveys to study the evolution of the distribution of life satisfaction in Europe from 1973 until 2014. With respect to previous literature, we update the analysis taking into account the most recent years. Attending to the possible interpretation of subjective well-being inequality measures as indicators of social cohesion, we focus on those countries that are at the front of the European integration process, that is, those belonging to the Eurozone when the common currency started to be in circulation: Austria, Belgium, Spain, East-Germany, West-Germany, Finland, France, Greece, Ireland, Italy, Luxembourg, The Netherlands, and Portugal (Euro-12).

The question formulated in most of the standard Eurobarometer surveys reads: "On the whole, are you very satisfied, fairly satisfied, not very satisfied or not at all satisfied with the life you lead?" The use of a verbal response scale makes especially disputable the computation of conventional inequality measures which are heavily dependent on the scales assigned. Thus, we use a median-based inequality measure that is scale independent and therefore appropriate for ordinal variables.

In particular we use the index proposed by Abul Naga and Yalcin (2008), which satisfies the Allison and Foster (2004) ordering. Besides, we demonstrate that it can be characterized as a measure of polarization for a particular range of the parameters. This property makes the Abul Naga and Yalcin (2008) class of indicators especially powerful to analyze the life satisfaction distribution dynamics. Madden (2011), Dutta and Foster (2013) and Balestra and Ruiz (2014) have applied previously this class of indicators to study inequality in subjective well-being data. We are the first to use the measure to study polarization. An interesting property of the ANY index is that it allows ascribing different weights for the probability mass points placed at both sides of the median. This property is of great interest in order to interpret the results.

Our analysis shows a number of interesting results. Firstly, at the Euro-12 aggregate level we observe that the life satisfaction inequality level has described a J-shaped curve between 1995 and 2014. Consequently, life satisfaction inequality was significantly higher at the end of the series. Secondly, previous aggregate finding is the result of different patterns at country level. We distinguish two different group of countries: on the one hand, those that show low and stable levels of life satisfaction inequality over the covered period: Austria, Belgium, East Germany, West Germany, Finland, France, Luxembourg, and The Netherlands; and on the other hand, those that show high and changing inequality levels: Spain, Greece, Ireland, Italy, and Portugal. Thirdly, we observe that the stability in life satisfaction inequality levels in the first group of countries is due to offsetting movements at both sides of the median of the distribution along the whole period. On the contrary, in the other group of countries and especially in the Mediterranean ones, what drives the overall trend are changes in the lower part of the distribution. Finally, as regards polarization, its evolution is quite similar to that of inequality, although some differing trends are observed in Portugal, Ireland and Luxembourg.

In the next section we define the concepts of subjective well-being and life satisfaction, and discuss several issues involved in the debate on their relevance for policy evaluation. In section 2 we also explain technical issues involved in the measurement of

inequality and polarization of ordinal variables. In section 3 we present and discuss main empirical findings. The paper ends with the conclusions.

2. Measuring inequality and polarization in subjective well-being

2.1. Subjective well-being and life satisfaction: definition and relevancy

Subjective experience of life, measured by means of large scale surveys, is arising as an essential component of well-being. But broad acceptance of subjective well-being measures notwithstanding, we owe to discuss the specific measure used in this research: life satisfaction self-assessments; and particularly, its accomplishment of the three properties required for a measure to be relevant for public policy: validity, reliability, and interpersonal comparability.

In social sciences "subjective well-being refers to all of the various types of evaluations, both positive and negative, that people make of their lives [...], events happening to them, their bodies and minds, and the circumstances in which they live" (Diener, 2006 p. 153). Subjective well-being has two main dimensions: on the one hand, the hedonic or affective dimension that comprises (non reflective) emotional reactions to life events, such as joy and sadness; and on the other hand, the reflective or cognitive dimension, which entails a cognitive process in which individuals might take into account, apart from current feelings, 1) other persons' conditions, 2) own past experience and 3) expectations of the future (Frey and Stutzer, 2002).[2]

Life satisfaction is one type of cognitive subjective well-being. It assesses individual's life as a whole and is supposed to reflect how does the individual feel, no matter if she would choose another kind of life (something measured by a meta-evaluation) or even she things that her life could be more meaningful (measured through questions about meaning and engagement). Thus, it could be understood as a summary or integrative measure of the affective subjective well-being in which the individual herself gives weights to each instant feeling (or affective balance) based on her preferences, expectations of the future, own past experience and social comparisons. Nonetheless, the scope of life satisfaction self-assessments seems to be larger. Baumeister et al. (2012), who find that meaning is far from being reducible to affective well-being, state that life satisfaction and meaning may capture the same underlying phenomenon.

As regards the validity of these measures, currently it is well established (see references in Diener et al., 2013; Blanchflower and Oswald, 2008; Kahneman and Krueger, 2006; Ferrer-i-Carbonell and Frijters, 2004; Frey and Stutzer, 2002; Easterlin, 1974) that subjective well-being self-assessments, including life satisfaction scores, correlate strongly with certain brain processes, expressions such as smiling, cortisol levels, and

[2] Subjective well-being and ill-being is the product of evolution. Humans share with other animals the hedonic or affective subjective well-being, although their affective systems have different levels of complexity. Affective feelings are signals that allow sensitive living beings to adapt behaviour and thus enlarge survival probabilities. In humans, reasoning capacity enables to improve the quality of that survival in the form of well-being (Damasio [2010] 2012; Heylighen y Bernheim, 2000; Veenhoven 2005).

cardiovascular behaviours; all which are thought to track positive moods. Additionally, they have been found to predict the likelihood of the onset of certain health conditions and, once those conditions have been diagnosed, their duration. They also predict important future actions and events such as committing suicide losing the job, giving up an activity, getting married or divorced, etc. Finally, life satisfaction self-assessments are consistent with other psychological measures such as depression, self-esteem, and self-reported mood. Finally, tests of validity that use as benchmark measure the evaluations of external observers, who either because of their close relationship with the individual or their professional skills are able to infer individual's subjective well-being from the observable indicators listed above, also support the validity of life satisfaction self-assessments (see references in Diener et al., 2013; Kahneman and Krueger, 2006; Easterlin, 1974).

As regards the reliability of these measures, which has to do with the stability of the responses over short periods of time in which underlying circumstances are more likely to have hardly change (Helliwell and Wang, 2012), there is evidence that transitory moods driven by trivial and contingent circumstances such as minor strokes of luck or soccer games results may have an effect on life satisfaction self-reports. Nonetheless, those effects seem to be small. It is estimated that around 75% of the variance of life satisfaction scales is due to long-term factors (see references in Diener et al., 2013). As regards the remaining effect of transitory moods or random variations, it is reasonable to assume that they average out in representative population samples. (In fact, empirical evidence shows a very high persistence in the life satisfaction distribution along time at country level).

Finally, the assumption on the interpersonal comparability of life satisfaction self-assessments is subject to discussion in the literature. Fleurbaey et al. (2009) argue that comparisons are senseless because life satisfaction self-assessments depend on each individual aspirations or frame of reference, in particular her past history and the circumstances of her group of reference, which are heterogeneous across individuals.

The issue of heterogeneous rating scales was already considered by Easterlin (1974), who contended that in any case meaningful comparisons can be carried out. He focused on the interest associated to their implications for prospective behaviour of the respondents. Nonetheless, apart from that practical foundation of life satisfaction interpersonal comparisons (which has to do with the usage of the life satisfaction distribution as a proxy of social cohesion), it is reasonable to confront the thesis that the dynamics of the aspirations of individuals makes it impossible to interpret the distribution of life satisfaction self-assessments as a distribution of a genuine well-being measure.

Thus, it is currently well established that there are at least two phenomena that have long lasting effects on subjective well-being: unemployment and severe health conditions, such as severe disability and chronic pain (Kahneman and Krueger, 2006; Diener et al., 2013). Besides, Helliwell and Wang (2012) argue that if no external factor had a long lasting effect on life satisfaction its distribution would not change across countries, as it is the case. On the other hand, and probably related to Helliwell and Wang (2012) argument, reference groups do not seem to be completely constrained to individuals belonging to the same social status, country, or even culture, but all

individuals seem to share a common opinion about what life satisfaction is[3]. This thesis corresponds with the observed socio-economic gradient on life satisfaction[4] (see references in Diener et al., 2013).

2.2. Inequality and polarization in life satisfaction

Life satisfaction of individuals is gauged by means of survey questions which answers are categorized in a number of different states. Responses have an ordinal structure: the higher the category the more the life satisfaction. Notwithstanding there is some evidence that supports a cardinal interpretation of life satisfaction measures (Ferrer-i-Carbonell and Frijters, 2004), in this work we adopt an ordinal interpretation of life satisfaction self-assessments, which is less restrictive. Thus, as currently most social scientists do, we assume that life satisfaction self-assessments are a positive monotonic transformation of an underlying phenomenon of interest (the type of cognitive subjective well-being described above). Besides, we assume that life satisfaction self-assessments are interpersonally and ordinally comparable. Therefore, contrary to the cardinal approach, we do not make any assumption as regards the relative difference between satisfaction answers, but only that all individuals do share the same interpretation of each possible answer (Ferrer-i-Carbonell and Frijters, 2004).

Most of the conventional inequality measures take the mean as a reference point to measure the dispersion of the distribution. The application of these measures to ordinal variables requires defining a scale, thus making the ranking of countries heavily dependent on this choice (Allison and Foster, 2004). To see this, consider the following example. Let f_x = [0.2, 0.2, 0.2, 0.2, 0.2] and f_y = [0.4, 0.1, 0.1, 0.1, 0.3] be two frequency distributions of subjective well-being. Applying the typical scale c = [1, 2, 3, 4, 5], we have μ_x (c) = 3 > μ_y (c) = 2.8. If we use instead the alternative scale c' = [1, 2, 3, 4, 50], μ_x (c') = 12 and μ_y (c') = 16.3. Consequently, the robustness and validity of inequality assessments using inequality measures that depend on the mean might be also affected by rescaling. Consider, for instance, the coefficient of variation of both distributions using the different scales. CV_x (c) = 0.47 < CV_y (c) = 0.61, while applying the alternative scale yields CV_x (c') = 1.58 > CV_y (c') = 1.35.

The limitations of conventional measures to assess inequality in categorical variables make room for alternative approaches based on order statistics[5]. In particular, the use of the median as a reference point seems to provide an adequate framework. The measure proposed by Blair and Lacy (2000) was one of the first attempts to obtain a median-based inequality index for ordinal variables. More recently, Allison and Foster (2004) developed a partial inequality ordering (thereafter AF-ordering), which ranks

[3] Additionally, in a world each day more interconnected it is reasonable to think that the weight of distant individuals might increase.

[4] Veenhoven (2005) explains the existence of the social gradient in life satisfaction from an evolutionary approach: there are human needs, for instance love, health and challenge that must be fulfil and subjective well-being signals the level of fulfillment. Life satisfaction depends on the gratification of needs, rather than on the perceived realization of wants. Habituation and social comparisons would operate only with respect to wants (e.g., income, material goods, and professional career).

[5] Within this class we find the interquartile range, which is scale dependent, but since it is based on the ranking of individuals, it is order preserving. The main problem of this measure is that it ignores a large amount of information contained between the two quartiles.

distributions according to their spread to the median. Most of the proposals to measure disparities in ordinal variables are based on this ordering[6] (Apouey, 2007; Abul-Naga and Yalcin, 2008; Kobus, 2014), which makes them invariant to the numerical scale.

There is some controversy about the phenomenon that median-based measures do actually capture. According to Zheng (2008), median-based inequality indices are indeed polarization measures. For cardinal variables, bipolarization is seen as a measure of the size of the middle class which focuses on the concentration of the population at the two tails[7] (see Foster and Wolfson, 2010). As we describe below, inequality indices for ordinal variables measure how concentrated the data is around the lower and the upper category. The most unequal distribution has, indeed, all the probability mass equally concentrated at the top and the bottom categories. Note however that, for categorical variables, the most unequal distribution can coincide with the most polarized one. This case is clearly observed for variables with only two categories. The variance, which is typically characterized as a measure of inequality, would be given by the sample size multiplied by the probability of the first category (the bottom one) and the probability of the second category (the top one). The dispersion would be then maximized when the probability is equally distributed in both categories, or in other words, when there are two poles of equal size.

The previous example points out that the concepts of polarization and inequality in ordinal variables are closely related. These two distributional aspects have been extensively studied in the case of cardinal variables, concluding that these two features of the distribution are also finely intertwined, while being theoretically and empirically different concepts. Indeed, bipolarization measures must satisfy two basic properties (Foster and Wolfson, 2010; Wang and Tsui, 2000), namely increased spread and increased bipolarity. To make a clear distinction between dispersion and polarization in ordinal variables, we make use of these two properties following the literature on polarization, and identify them with their corresponding orderings for categorical variables.

Property 1 (Increased spread). A regressive transfer in the Pigou-Dalton sense from someone below the median to an individual above the median would increase bipolarization.

For categorical variables, the transfer is characterized as a *Median Preserving Spread* and this property is represented by the AF-partial ordering given in Definition 1.
Let $F(x) = (F_1, \ldots, F_{J-1}, 1)$ be the cumulative distribution of the categorical variable x and $f(x) = (f_1, \ldots, f_{J-1}, f_J)$ the probability distribution function, where f_j is the proportion of population for the category j. Let m denote the median category such that $F_{m-1} < 1/2$ and $F_m \geq 1/2$.

Definition 1. AF-ordering (Allison and Foster, 2004). Given two cumulative distribution functions F^1 and F^2, F^2 has greater spread from the median than F^1, that is $F^1 \prec_{AF} F^2$ if the following conditions are satisfied:

[6] See Cowell and Flachaire (2012) and Silber and Yalonetzky (2012) for alternative measures of inequality in ordinal variables.
[7] The concept of bipolarization is associated with the population in two poles, while polarization considers a clustering of population around an arbitrary number of poles. While being pretty close definitions both phenomena can present contrasting evolutions (see Duclos and Taptué, 2014).

(1) F^1 and F^2 have the same median category, $m(F^1) = m(F^2) = m$.
(2) $F_j^1 \leq F_j^2 \quad \forall\, j < m$.
(3) $F_j^1 \geq F_j^2 \quad \forall\, j \geq m$.

F^1 has a greater proportion of population concentrated around the median and consequently, F^1 is less spread away from the median. In this context, F^1 is associated with lower levels of inequality for all possible scales.
To see this graphically, in Figure 1 we present the distribution of a life satisfaction variable with five possible categories: "not at all satisfied", "not very satisfied", "neither satisfied nor dissatisfied", "fairly satisfied", "very satisfied". For convenience we assume that individuals are clustered in two categories "not very satisfied" and "fairly satisfied", above and below the median category: "neither satisfied nor dissatisfied". Now assume that there is a relocation of the groups. In the left panel, individuals that were "not very satisfied" become "not at all satisfied" and the group of population that was "fairly satisfied", becomes "very satisfied". These movements increase the spread of the distribution from the median, and consequently both bipolarization and inequality would increase.

Property 2. Increased bipolarity (IB). A progressive Pigou-Dalton transfer on either side of the median would increase polarization. The transfer decreases the dispersion within the group of population below/above the median, thus fostering the levels of polarization, while decreasing inequality.

Definition 2. Increased bipolarity (IB) transfer. We say the there is an increased bipolarity transfer of a proportion $\zeta > 0$, from the category j to j+1 and from k to k-1 if $f_j, f_k > \zeta$, $j+1 \leq k\text{-}1$ and $j \geq m$ or $k < m$.

Definition 3. Increased bipolarity ordering (Apouey, 2007). Let $\succ_{IB}$ define the partial ordering, for any two distributions F^1, F^2, such that $m(F^1) = m(F^2) = m$, we say that $F^1 \prec_{IB} F^2$ if F^2 is obtained from F^1 by a finite sequence of IB transfers.

Figure 1. Increased spread from the median (left) and increased bipolarity (right) in life satisfaction

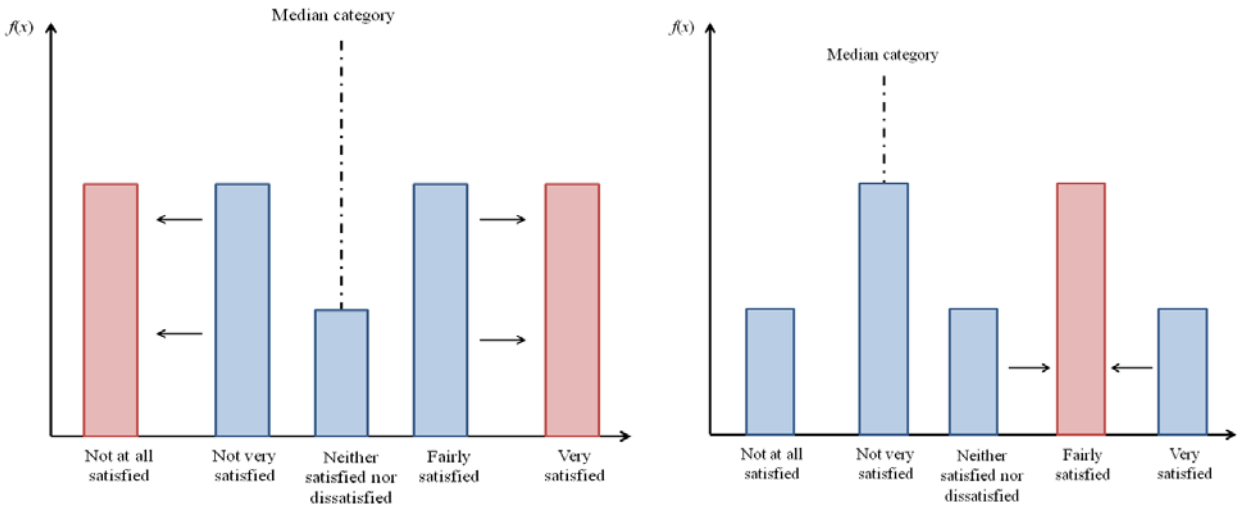


Figure 1 (right panel) exemplifies graphically an increase of bipolarity in the case of categorical variables. Below the median all individuals are "not very satisfied" with their life, while at the median and above, there is an equal proportion of individuals that are "satisfied" and "very satisfied". Suppose that these two groups cluster at the category "fairly satisfied" as indicated in Figure 1. This new situation would clearly present lower levels of inequality since the distribution above the median is more homogeneous, however polarization would rise. This is because the new situation is reached through two movements of the probability mass: an increased spread from the median (from "satisfied" to "fairly satisfied") and a decreased spread (from "very satisfied" to "fairly satisfied"). Consequently, the first movement of population must receive more weight than the second one when measuring polarization.

The inequality measures developed in the literature (Blair and Lacy, 2000; Allison and Foster, 2004; Abul Naga and Yalcin, 2008) focused on the partial AF-ordering, but polarization measures should verify also IB. Indeed, IB property distinguishes bipolarization from inequality, since a progressive transfer between individuals that are both above or below the median would decrease inequality while increasing polarization (Foster and Wolfson, 2010).

Abul Naga and Yalcin (2008) relied solely on the partial AF-ordering to construct a family of inequality indices which is invariant to scale.

Let $\Phi(F)$ be an order-preserving function of the $\succ_{AF}$ relation. Let $\hat{F}$ denote the most egalitarian distribution, which has all the probability mass concentrated at the median category. $\tilde{F}$ is the most unequal distribution, where half of individuals are classified in the lowest category and the other half of the population is placed at the best possible state[8]. The family proposed by Abul Naga and Yalcin (2008) is constructed by normalizing $\Phi(F)$ by $\Phi(\hat{F})$ and $\Phi(\tilde{F})$, as follows:

$$I(F)=\frac{\Phi(F)-\Phi(\hat{F})}{\Phi(\tilde{F})-\Phi(\hat{F})}. \qquad (1)$$

A function of the form $\Phi(F)=\sum_{j<m} g_1(F_j)-\sum_{j\geq m} g_2(F_j)$ was chosen, which satisfies the AF-ordering when g_1 and g_2 are strictly increasing on F (see Lemma 4, Abul Naga and Yalcin (2008)). Replacing $g_1(F_j)=\left(F_j\right)^{\alpha}$ and $g_2(F_j)=\left(F_j\right)^{\beta}$ in Eq. (1), the ANY index can be written as,

$$I_{\alpha,\beta}(F)=\frac{\sum_{j<m}(F_j)^{\alpha}-\sum_{j>m}(F_j)^{\beta}+(n+1-m)}{\kappa_{\alpha,\beta}+(n+1-m)},$$

[8] The most equal distribution has all the probability mass in one category and then we have: $\hat{F}=[0,0,...,1,..1]$, $\hat{f}=[0,0,...,1,...0]$. Note that $\hat{F}$ is not unique. The probability mass can cluster around any category, so there are *C* different most equal distributions. The most unequal distribution concentrates the probability mass around the two ends: $\tilde{F}=[0.5,0.5,...,1]$. $\tilde{f}=[0.5,0,...,0,...,0.5]$.

where $\kappa_{\alpha,\beta} = (m-1)(1/2)^{\alpha} - (1+(n-m)(1/2)^{\beta})$. α and β are parameters to represent the value judgments of the society for inequalities below and above the median. $I_{\alpha,\beta}$ satisfies the AF-ordering for any positive value of parameters α and β (see Lemma 4 in Abul Naga and Yalcin (2008)). However, the range of the parameters was restricted for simplicity and the measure was originally defined for parameter values higher or equal to 1. According to Abul Naga and Yalcin (2008), the combination of convex (parameter values higher than 1) and concave (parameter values between 0 and 1) functions would make it difficult to determine the sensitivity of the index to specific parts of the distribution. It should be, however, noted that this choice on the parameters has specific implications in terms of the consideration of IB movements. The following proposition characterizes the ANY measure as an index of polarization.

Proposition 1. $I_{\alpha,\beta}$ satisfies the partial IB-ordering for $\alpha > 1$, $\beta \in]0,1[$.

Proof. See Appendix.

Then, for parameter values strictly higher than one, ANY measure increases due to IB movements below the median, while decreasing if this kind of transfers takes place above the median. For the limiting case given by the so-called absolute index ($\beta = \alpha = 1$), IB movements lead the measure unchanged.

Let us illustrate the behavior of this measure with an example. Table 1 presents three hypothetical distributions of seven categories which share a common median (the fourth category). F_2 and F_3 are obtained from F_1 by an IB movement below and above the median respectively. $A(0.5)$ refers to the Apouey (2007) measure which is an index of polarization for ordinal variables which satisfies both the AF- and the IB-ordering. We observe that this measure reports an increase in polarization of the same amount, given that it is characterized by the property of symmetry. $I_{2,2}$ denotes the ANY measure when both parameters are set to two. We observe that the clustering below the median increases the value of this measure while the IB movement above the median decreases inequality. While it would be debatable whether inequality measures in categorical variables should be consistent with the IB ordering, the choice must be congruent for all parts of the distribution. $I_{2,0.5}$ characterizes ANY measure as a polarization index, in the sense that it increases due to IB movements above and below the median. In contrast, $I_{0.5,\ 2}$ characterizes ANY measure as an inequality index. Since an IB transfer would make the distribution more homogeneous in either side of the median so inequality would be reduced within that part of the distribution and hence overall inequality also falls. This example highlights the flexibility of the ANY measure, not only because it allows us to ascribe different weights to differences below and above the median, but also because we can measure distinct distributional phenomena depending on the value of the parameters.

The ANY polarization measure becomes more sensitive to changes at the top of the distribution as α tends to ∞ for a given value of β. In the limit we have,

$$I_{\infty,\beta}(F) = \frac{-\sum_{j \geq m}(F_j)^{\beta} + (n+1-m)}{(-1+(n-m)(1/2)^{\beta} + (n+1-m)}, \beta \in]0,1[.$$

Table 1. Sensitivity of ANY measure to IB movements

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	$A(0.5)$	$I_{2,2}$	$I_{2,0.5}$	$I_{0.5,2}$
f_1	0.15	0.1	0.15	0.2	0.15	0.1	0.15				
F_1	0.15	0.25	0.4	0.6	0.75	0.85	1	0.3363	0.5333	0.4190	0.6576
f_2	0.05	0.3	0.05	0.2	0.15	0.1	0.15				
F_2	0.05	0.35	0.4	0.6	0.75	0.85	1	0.3442	0.5467	0.4436	0.6411
f_3	0.15	0.1	0.15	0.2	0.05	0.3	0.05				
F_3	0.15	0.25	0.4	0.6	0.65	0.95	1	0.3442	0.5200	0.4233	0.6485

On the other hand, ANY polarization measure becomes less sensitive to changes in the upper tail as β tends to 0. The limiting case would be characterized by the following index:

$$I_{\alpha,0}(F)=\frac{\sum_{j<m}(F_j)^{\alpha}+(n+1-2m)}{(m-1)(1/2)^{\alpha}},\alpha>1.$$

The ANY polarization measure can be expressed as a weighted sum of the previous two cases as follows,

$$I_{\alpha,\beta}=\omega_1 I_{\alpha,0}+\omega_2 I_{\infty,\beta}-\left(\frac{n+1-2m}{\kappa_{\alpha,\beta}+(n+1-m)}\right), \qquad (2)$$

where $\omega_1=\dfrac{\kappa_{\alpha,0}+(n+1-m)}{\kappa_{\alpha,\beta}+(n+1-m)},\omega_2=\dfrac{\kappa_{\infty,\beta}+(n+1-m)}{\kappa_{\alpha,\beta}+(n+1-m)}.$

ANY index was proposed as a measure of inequality in ordinal variables, being characterized by a number of desirable axioms that an inequality index should satisfy (Abul Naga and Yalcin, 2008). Using Proposition 1, we can characterize the ANY measure as an inequality index when $\alpha\in\left]0,1\right[,\beta>1,$ in the sense that an IB ordering is not satisfied. Then, for this specific range of the parameters, an increase in bipolarity on either side of the median would reduce inequality levels in that part of the distribution and consequently, overall disparities would also decrease.

The inequality measure is more sensitive to the top of the distribution as α tends to 0 for a given value of β. the limit when $\alpha\to 0$, abstracts from the distribution below the median,

$$I_{0,\beta}(F)=\frac{n-\sum_{j\geq m}(F_j)^{\beta}}{n-1-(n-m)(1/2)^{\beta}},\beta\geq 1.$$

ANY inequality measure becomes less sensitive to dispersion above the median as β tends to infinity. The limiting case is given by the following index:

$$I_{\alpha,\infty}(F) = \frac{\sum_{j<m}(F_j)^{\alpha} - t + (n+1-m)}{(m-1)(1/2)^{\alpha} + n - m}, \alpha \in]0,1[,$$

where t are the number of categories such that $F_j = 1$.

Analogously, the ANY inequality measure can be also expressed as a weighted sum of the of the two limiting cases,

$$I_{\alpha,\beta} = \omega_1 I_{\alpha,\infty} + \omega_2 I_{0,\beta} - \left(\frac{n-t}{\kappa_{\alpha,\beta} + (n+1-k)} \right), \quad (3)$$

where $\omega_1 = \frac{\kappa_{\alpha,\infty} + (n+1-k)}{\kappa_{\alpha,\beta} + (n+1-k)}, \omega_2 = \frac{\kappa_{0,\beta} + (n+1-k)}{\kappa_{\alpha,\beta} + (n+1-k)}.$

3. Distributional dynamics of life satisfaction in Europe

3.1. Data

Life satisfaction is measured in the Eurobarometer surveys since 1975 by means of the question: "on the whole, are you very satisfied, fairly satisfied, not very satisfied or not at all satisfied with the life you lead?," whose wording has remained unchanged over the whole period (1975-2014), which is essential to make the measure perfectly comparable on this respect[9]. Diener (2006) distinguishes two possible meanings regarding *life* in the life satisfaction question: it may refer either to *current* life-as-a-whole, or to life *since birth*. The wording of the question in the Eurobarometer surveys ("[...] the life you lead […]") points to the former meaning, although not explicitly. The measurement scale is not numerical, what further supports an ordinal approach. A nice feature of non numerical scales is that they avoid individuals focusing at either extremes or the midpoint of the scale.

In order to have a more homogeneous group of countries and increase the relevancy of our results through possible policy implications, we limit our sample to those countries belonging to the Eurozone in 2002, when the common currency started to be in circulation. These countries are often referred to as the Euro-12 group of countries; although in our case we split Germany into its two historical entities. Seven countries: Belgium, West Germany, France, Ireland, Italy, Luxembourg, and The Netherlands have participated in all Eurobarometer surveys. Greece joint the survey in 1980; Spain and Portugal in 1985; East Germany in 1990; and finally, Austria and Finland in 1995.

[9] Actually, since 2012 the survey distinguishes among individuals who do not answer the question and those that answer "I do not know", but given that only very few people choose this response category it does not seem a problem to simply omit them.

A technical issue to be considered is the ordering of the questions in the survey, which is known to affect life satisfaction self-assessments. Most of the Eurobarometer surveys formulate the life satisfaction question at the beginning of the interview to avoid this problem. The answers of subjective surveys may be also shaped by the social nature of these surveys. In particular, as regards life satisfaction self-assessments it is observed that individuals especially predisposed to conform with social norms are less likely to report low levels of life satisfaction (see references in Diener et al., 2013; Easterlin, 1974). The fact that Eurobarometer interviews are conducted face-to-face makes its data more prone to this source of bias, therefore we should keep this caveat in mind when interpreting results.

Eurobarometer surveys usually targets 1,000 persons per country[10]. Country level measures are computed using national weights provided in the Eurobarometer surveys databases, and measures at the Euro-12 aggregate level transforms national weights according to the population size of the country. In most of the surveys that collect data on life satisfaction the response rate is remarkably high: always over 98%.

3.2. Results

Figure 2 displays the evolution of life satisfaction inequality between 1995 and 2014 inequality for the different Euro-12 countries and for this group of countries as a whole. We have calculated the two limiting cases: when only the bottom part is considered ($\alpha \to 0$, $\beta = 1.5$) and when only changes above the median affect to inequality ($\alpha = 0.5$, $\beta \to \infty$); and an intermediate case given by $\alpha = 0.5$, $\beta = 1.5$.

At the Euro-12 aggregate level, we observe that inequality remained stable during first years, fell sharply in 1999 and then started to rise erratically, reaching the initial level and stabilising around 2004. Since 2008, coinciding with the coming up of the Great Recession, it is observed a steady upward trend in life satisfaction inequality. As a result, in the Euro-12 group of countries life satisfaction inequality is much higher in 2014 than it was in 1995. Changes in the whole distribution are mainly driven by trends in its lower part, especially during the last decade.

Aggregate movements may hide divergent tendencies at country level whose analysis is crucial to have a complete picture of the phenomenon under study and hence be able to derive possible policy implications. Basically, it is important to find out whether life satisfaction inequality has grown in all countries or the aggregate outcome has been driven by changes in the life satisfaction distribution of some specific countries. The analysis of the period 1995-2014 can be readily subsumed in the analysis of country level longer series, which besides provides interesting additional information. First, we will comment each entire series, and then gather together the findings that explain the evolution of life satisfaction inequality at the Euro-12 aggregate level over the last 20 years.

[10] With the exception of Luxembourg that usually collects data of 300 individuals.

Figure 2. Abul Naga-Yalcin inequality index for selected values of the parameters.

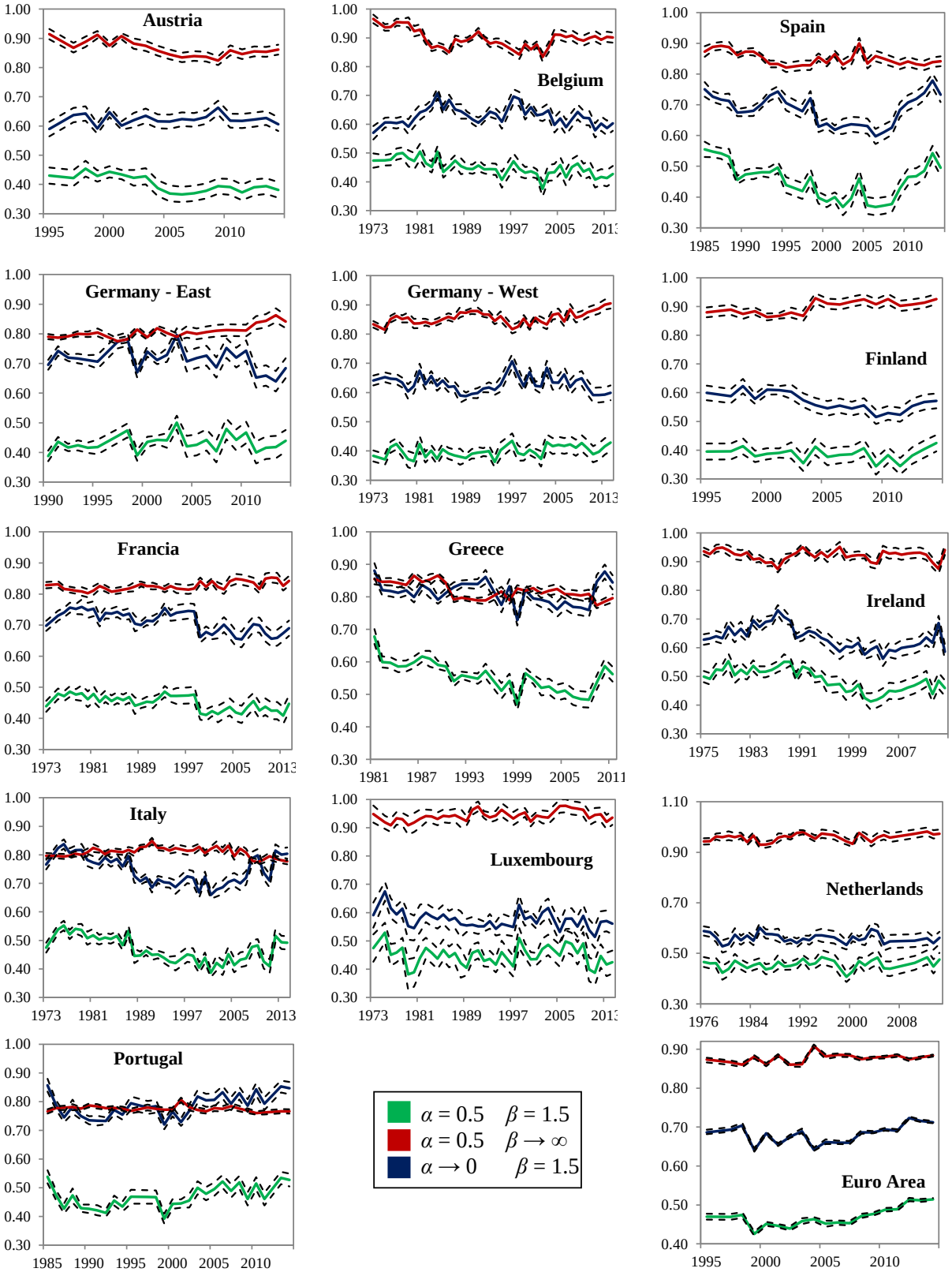
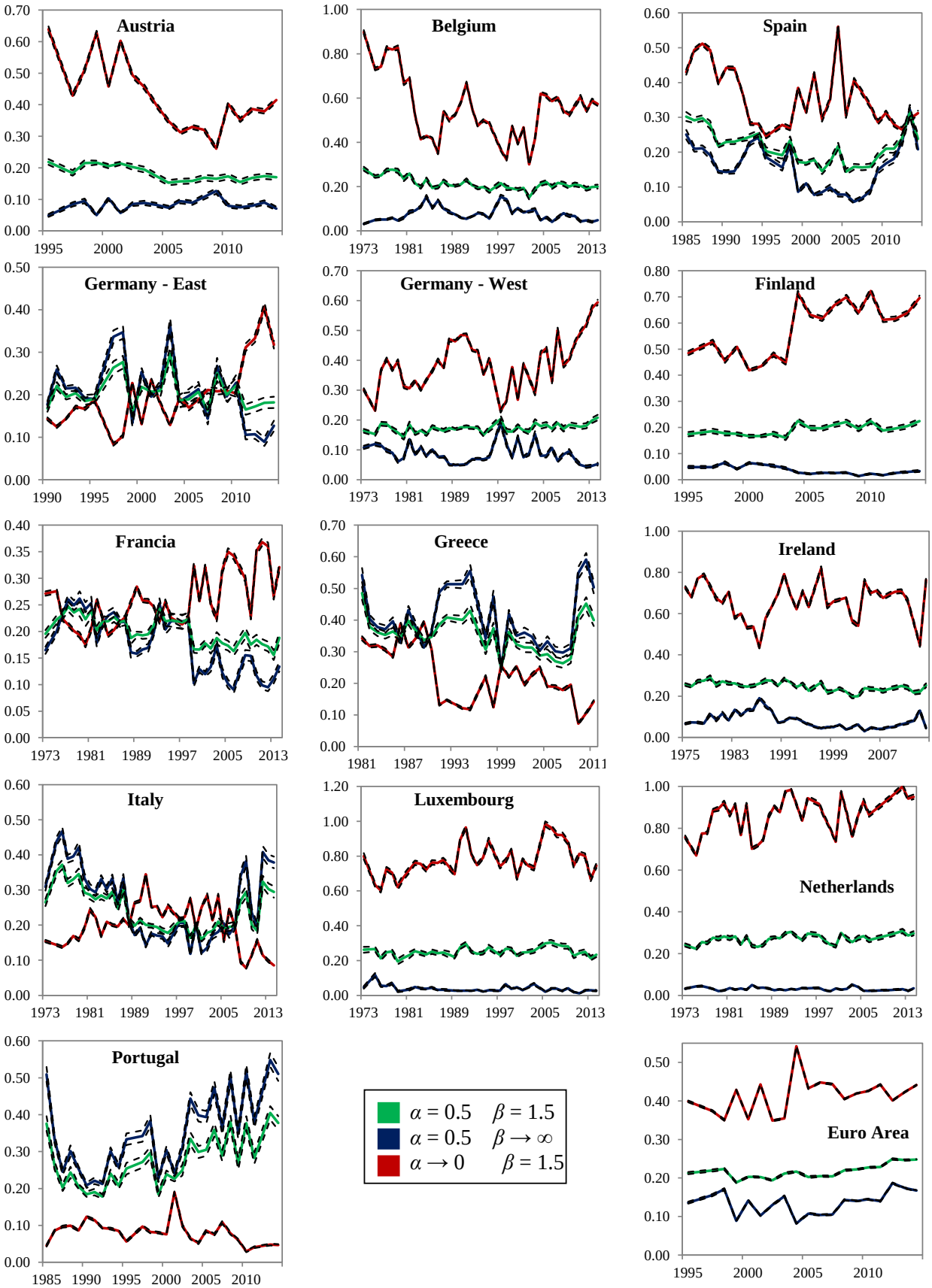


Figure 3. Abul Naga-Yalcin polarization index for selected values of the parameters.



All countries, with the only exception of Greece, present rather similar levels of life satisfaction inequality at the beginning and at the end of their respective series[11]. Nevertheless, these levels and their evolution over each period differ remarkably. Thus, two groups of countries can be distinguished: on the one hand, those that show low and quite stable levels of inequality along the covered period: Austria, Belgium, East Germany, West Germany, Finland, France, Luxembourg, and The Netherlands; and on the other hand, those that show high and changing inequality levels: Spain, Greece, Ireland, Italy, and Portugal.

It must be noted that contrary to what has been found by previous research (Veenhoven, 2011; Becchetti *et al.*, 2014), West Germany is among the countries that display stable levels of inequality over the period. This divergent finding is not only explained by the usage of different samples and response variables, given that Veehoven (2011) works also with the Eurobarometer Surveys, but it would be mainly explained by the different indicators used to measure disparities in life satisfaction. Thus, previous findings were obtained by means of standard inequality measures that require in the first place to cardinalize the life satisfaction variable, while our estimation respects its ordinal nature.

An interesting finding regards the fact that the observed stability in life satisfaction inequality in the first group of countries reflects offsetting movements observed along the whole series at both sides of the median of the distribution. Thus, contrary to what is observed in the other group of countries in this group the upper part of the distribution is as sensitive as the lower part to the different determinants of the life satisfaction distribution. The only clear divergence that avoid talking about mirroring patterns at both sides of the median is the higher volatility observed in the lower part of the distribution, which is less evident in Austria and Finland.

Among the countries that present high and changing levels of inequality along the period, Spain and Greece show a common pattern: first, a downward trend since their entrance into the European Economic Community (EEC) until 2007, and then a sharp rise in the inequality level that virtually cancelled out previous reduction. No doubt the Great Recession has supposed a dramatic change in the evolution of the life satisfaction distribution in both countries

The analysis of the changes at both sides of the median of the distribution shows that the evolution of the whole distribution is basically determined by the trend observed in its lower part. Inequality in the upper part of the distribution arises rather stable, although some significant departures should be noted. Thus, at the beginning of the 1990s, in both countries, movements in the upper part of the distribution compensated the movements observed in its lower part. In the case of Spain, it is also remarkable that in the mid-2000 there was a brief boost in inequality driven by an increase in the spread of the upper part of the distribution. In the case of Greece, a slight movement toward the median in the upper part of the distribution during the Great Recession has mitigated its effect on inequality[12].

[11] Inequality in Greece was higher at the beginning of the 1980s than now.

[12] In Greece inequality in the upper part of the distribution is significantly lower at the end of the series than at the beginning, being this crucial to explain the fact that inequality in the whole distribution is also lower in 2014 than in 1981.

In contrast to Spain and Greece, Portugal, Italy and Ireland, which have also endured very severely the socio-economic turmoil associated to the Great Recession, do not seem to have experienced the financial crisis as a turning point in the evolution of their respective life satisfaction distributions. Indeed, in these three countries we observe an upward trend in life satisfaction inequality levels that started before the Great Recession.

Portugal, after a sharp decrease in life satisfaction inequality during first years after entering the EEC, has experienced an erratic upward trend in life satisfaction inequality since the beginning of the 1990s, with just a slight decrease in inequality at the end of that decade. Interestingly, after a clear rise in inequality along the 2000s, the level of inequality stabilized around 2007, coinciding with the coming up of the Great Recession. As was the case of Spain and Greece, the evolution of the whole distribution is basically determined by the trend observed in its lower part.

In Italy the upward trend in life satisfaction inequality started at the beginning of the 2000s after having been decreasing steadily since the mid-70s. The same pattern observed in the other Mediterranean countries as regards the movements of inequality above and below the median applies: inequality in the upper part of the distribution remains fairly constant over the period, and trends in its lower part determine changes in the whole distribution.

In Ireland inequality remained roughly constant during the 70s and 80s and decreased rather sharply during the 1990s. At the beginning of the 2000s, the previous trend ceased and it seems to have reversed, although the level of inequality at the end of the series is not statistically different from the one observed when the series reached its lowest level at the beginning of the 2000s. As regards movements at both sides of the median of the distribution, in Ireland its lower and upper parts presented opposite trends during the 70s, 80s and first 90s that offset each other, thus explaining the overall stability in life satisfaction inequality during those years. Along the 90s and up to the start of the Great Recession inequality in the upper part of the distribution held constant. Therefore, both the overall decrease in life satisfaction inequality during the 90s, and the ascending trend observed since the beginning of the 2000s were driven by changes in the lower part of the distribution. During the Great Recession departures from the median have continued in the lower part of the distribution while in the upper part the tendency has been the opposite. As result, overall inequality has remained fairly stable during the crisis, and is not statistically higher in 2014 than it was when it reached its lowest level at the beginning of the 2000s.

Returning to the evolution of life satisfaction inequality since 1995 until 2014 at the Euro-12 aggregate level, the first remarkable change in the distribution: the sharp fall in life satisfaction inequality in 1999 is the product of the steady downward trend in life satisfaction inequality in Spain, Greece, Ireland and Italy; the fact that Portugal experienced a short fall in inequality that year; and the fact that countries that display very stable levels of inequality over the period under study experienced, nonetheless, a short fall in those levels in 1999, especially East Germany, France, and The Netherlands. As regards the other remarkable change in the distribution: the steady increase in life satisfaction inequality since 2008, it seems to be basically the product of the upward trends observed in Ireland, Italy, and, especially, Spain, and Greece.

The rest of this section focuses on the evolution of polarization and the main divergences observed between this phenomenon and inequality. Figure 3 presents the evolution of the ANY index as a polarization measure. In order to compare these results with the time trend of inequality in life satisfaction, we have computed the two limiting cases: when only the bottom part is considered ($\alpha = 1.5$, $\beta = 0$) and when only changes above the median affect to polarization ($\alpha = \infty$, $\beta = 0.5$); and an intermediate case $\alpha = 1.5$, $\beta = 0.5$. Little differences can be found between the two limiting measures for polarization and inequality. While presenting more pronounced variations at the top of the distribution in the case of polarization, the evolution of both phenomena is essentially the same. If the whole distribution is considered, we can observe some differing patterns for some particular countries.

In Portugal, the second half of the 90s is characterized by a stagnation phase in the case of inequality, while polarization presents an ascending trend. The difference is driven by IB movements at the top of the distribution, which leads an increase of polarization above the median that fosters the raise of the overall measure. In Ireland, an ascending trend is observed along the 2000s in the case of inequality due to the transfer of probability mass from the median to the bottom. This movement increases the spread of the distribution below the median, thus fostering both inequality and polarization. At the top of the distribution, we observe stable levels of inequality, while polarization presents a decreasing trend driven by a reduction of bipolarity. Therefore, in the case of inequality, the increase of the spread at the bottom of the distribution leads an increase in inequality in life satisfaction. In contrast, the decrease in polarization levels above the median offset the increase in the spread at the bottom and hence polarization remained roughly constant during this decade. Luxembourg presents a decrease of inequality from 1975 to 1980, driven by a reduction in the spread at both sides of the median, especially at the bottom. These movements also involve an increase of the concentration above the median, which increases bipolarity and hence dampens the decrease of polarization levels.

4. Conclusions

We have analysed the evolution of life satisfaction inequality and polarization in the Euro-12 group of countries, that is, those countries that have belonged to the Eurozone since the common currency started to be in circulation, and furthermore, by 2002 had already a more or less long trajectory in the European integration process. In order to study inequality and polarization in life satisfaction we have employed an ordinal measure that respects the ordinal nature of this variable. In particular we have used the index proposed by Abul Naga and Yalcin (2008), which satisfies the Allison and Foster (2004) ordering. Besides, we have demonstrated that it also satisfies the increased bipolarity ordering (Apouey, 2007) for a particular range of the parameters, thus being characterized as a measure of polarization.

The main finding is that life satisfaction inequality was significantly higher in 2014 (0.54) than in 1995 (0.49), in particular there has been around a 10% increase in the level of inequality over the period. Otherwise, inequality has not grown evenly across Euro-12 countries, but the rise at the aggregate level is basically explained by the increase in inequality in the Mediterranean countries and Ireland. The underlying reason is that in these countries sizable shares of the sample have moved away from the median in the lower part of the distribution, having remained its upper part quite stable. In

general, we find that in the Mediterranean countries and Ireland, contrary to what is observed in other countries, the upper part of the distribution is much less responsive than its lower part, especially in last years. As regards the polarization phenomenon, its evolution over time is quite similar to that of inequality, albeit some deviances are observed in Portugal, Luxembourg and Ireland.

At the light of this findings, and if we agree that "happiness convergence could also be […] a useful barometer of changes in the political landscape, societal values, and citizens' sentiments about further European Integration" (Apergis and Georgellis, 2015, p. 68), our analysis evidences, coinciding with the sentiment instilled by the media these days, the gloomy present of the European integration process. Although it must be highlighted that main social tensions arise within country, and not between countries, because an important share of the population (the share located in the upper part of the life satisfaction distribution) of those countries with increasing levels of inequality is not responsive to the forces that affect the lower part of the distribution.

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6. Appendix

Proof of Proposition 1

(1) IB movement above the median.

For any proportions (f_a, f_b, ζ); f_a, f_b, $\zeta \geq 0$, $f_a \geq \xi$, $f_a + f_b + \zeta \leq 1/2$ and $f_b + \xi > 0$ and for any two distributions F^1, F^2, such that,

$$m(F^1) = m(F^2)\ ;\ m \in [1, J-2],$$

$$F_j^1 = F_j^2 \text{ for } j \in [1, J-3] \text{ if } J \geq 4$$

$$F_{J-2}^1 = \frac{1}{2} + f_a \quad \text{and} \quad F_{J-1}^1 = \frac{1}{2} + f_a + f_b$$

If F^2 is obtained from F^1 by an IB transfer, setting j = J-2 and k = J, we have:

$$F_{J-2}^2 = \frac{1}{2} + f_a - \xi \quad \text{and} \quad F_{J-1}^1 = \frac{1}{2} + f_a + f_b + \xi$$

Then, $I_{\alpha,\beta}(F^1) < I_{\alpha,\beta}(F^2)$ if and only if $\Phi(F^1) < \Phi(F^2)$. Then,

$$\sum_{j<m} g_1(F^1) - \sum_{j\geq m} g_2(F^1) < \sum_{j<m} g_1(F^2) - \sum_{j\geq m} g_2(F^2).$$

Since $\sum_{j<m} g_1(F^1) = \sum_{j<m} g_1(F^2)$,

$$-\sum_{j\geq m} g_2(F^1) < -\sum_{j\geq m} g_2(F^2) \rightarrow \sum_{j\geq m} g_2(F^1) > \sum_{j\geq m} g_2(F^2)$$

This is equivalent to,

$$g_2(f_a) + g_2(f_a + f_b) > g_2(f_a - \xi) + g_2(f_a + f_b + \xi)$$

$$g_2(f_a) - g_2(f_a - \xi) > g_2(f_a + f_b + \xi) - g_2(f_a + f_b)$$

The previous statement is true if $g_2(F)$ is strictly concave on $\Re_+$. Taking $g_2(F) = (F_j)^\beta$, the previous result implies that IB ordering is satisfied if $\beta \in (0,1)$. □

(2) IB movement below the median.

For any proportions (f_a, f_b, ζ); f_a, f_b, $\zeta \geq 0$, a $\geq \xi$, $f_a + f_b + \xi \leq 1/2$ and $f_b + \xi > 0$ and for any two distributions F^1, F^2, such that,

$$m(F_1) = m(F_2)\ ;\ j \in [3, J],$$

$$F_j^1 = F_j^2 \text{ for } j \in [4, J]\ ;\ J \geq 4$$

$$F_1^1 = f_a \quad \text{and} \quad F_2^1 = f_a + f_b$$

If F^2 is obtained from F^1 by an IB transfer, setting j = 1 and k = 3, we have:

$$F_1^2 = f_a - \xi \quad \text{and} \quad F_2^1 = f_a + f_b + \xi$$

Again, $I_{\alpha,\beta}(F^1) < I_{\alpha,\beta}(F^2)$ if and only if $\Phi(F^1) < \Phi(F^2)$. Then,

$$\sum_{j<m} g_1(F^1) - \sum_{j\geq m} g_2(F^1) < \sum_{j<m} g_1(F^2) - \sum_{j\geq m} g_2(F^2).$$

Given that $\sum_{j\geq m} g_2(F^1) = \sum_{j\geq m} g_2(F^2)$, this is equivalent to

$$\sum_{j<m} g_1(F^1) < \sum_{j<m} g_1(F^2),$$

which is satisfied if,

$$g_1(f_a) + g_1(f_a + f_b) < g_1(f_a - \xi) + g_1(f_a + f_b + \xi)$$

$$g_1(f_a) - g_1(f_a - \xi) < g_1(f_a + f_b + \xi) - g_1(f_a + f_b)$$

The previous statement is true if $g_1(F)$ is strictly convex on $\Re_+$. Taking $g_1(F) = \left(F_j\right)^{\alpha}$, the previous result implies that IB ordering is satisfied if $\alpha > 1$. □